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VIRTUAL MANIPULATIVES AND INSTRUCTIONAL STRATEGIES FOR TEACHING MATHEMATICAL CONCEPTS AND SKILLS TO STUDENTS WITH SPECIAL NEEDS

Review Study

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Abstract

Students with special needs may face difficulties in demonstrating abstract concepts and the skills associated with these concepts. These challenges can vary based on individual differences, learning characteristics, types of disabilities, and many other factors. Manipulatives serve to concretize abstract concepts and skills, allowing students to learn through seeing, hearing, and doing. In the special education literature, there are two types of manipulatives: concrete and virtual manipulatives. With the advancement of technology, it is suggested that virtual manipulatives can be effective in teaching mathematical concepts and skills to students with special needs. Virtual manipulatives can be beneficial for both practitioners and students. This study examines manipulatives, virtual manipulatives, and the instructional processes involved in imparting mathematical concepts and skills to students with special needs, as well as the related research in this context. Additionally, virtual manipulative platforms that practitioners can use are introduced.

Keywords: Special needs, Mathematics, virtual manipulatives, virtual-representational-abstract strategy

1. Introduction

Technological changes and advancements are rapidly increasing today. As a result, our use of technologies to meet various needs has also risen. In the field of education, technology usage has almost become mandatory. Raja and Nagasubramani (2018) have stated that the positive aspects of technology can be utilized and that countries should implement technology-equipped educational processes. Therefore, it can be said that current technologies should be utilized to fully facilitate the processes of accessing and sharing educational information.

From early childhood education to higher education, various technologies are used in the education of students with special needs, just as they are used in all types and levels of education. Research focusing on the educational outcomes of these uses based on different variables is being conducted and reported. Studies in the literature focus on the use and effects of technologies combined with various instructional strategies to teach concepts and skills to students with disabilities such as intellectual disability, autism spectrum disorder, and learning disabilities. One of the technologies that related research focuses on, which can provide positive gains in application processes, is virtual manipulatives that require the combined use of digital tools with applications or web-based platforms. The effectiveness of virtual manipulatives in imparting mathematical concepts and skills is being researched and reported.

More research is needed to implement technology-equipped educational processes and to integrate technology-based instruction with reported positive outcomes, such as virtual manipulatives, into educational processes. Therefore, this study focuses on the use of virtual manipulatives within the relevant literature framework to impart mathematical concepts and skills to students with special needs. The study addresses concrete manipulatives and virtual manipulatives in the context of teaching mathematical concepts and skills, instructional strategies, the impacts of various studies in the literature, and virtual manipulative platforms that can be used in application processes. Since virtual manipulatives are computer-based simulations of concrete manipulatives (Bouck & Flanagan, 2010), concrete manipulatives are addressed first.

2. Concrete Manipulatives

Concrete manipulatives are defined as "physical artifacts that can be concretely handled by students and offer a large and deep set of sensory experience." (Bartolini & Martignone, 2020). Mathematics inherently involves abstract concepts (Yaşa & Kale, 2022). It can be said that manipulatives support students in making sense of mathematical concepts and skills by serving to concretize abstract concepts. In this context, the use of manipulatives is justified by the saying "*I hear and I forget, I see and I remember, I do and I understand*" (Marshall & Swan, 2008). Thus, manipulatives help students develop an abstract mathematical understanding through concretization (Hartshorn & Boren, 1990).

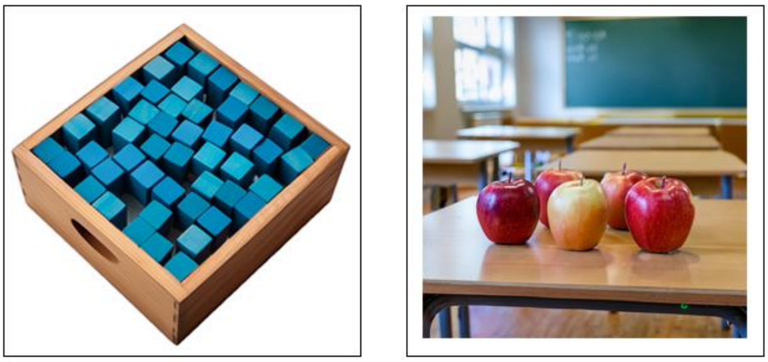

Figure 1. Concrete manipulatives*

As seen in Figure 1, concrete manipulatives are easily obtainable objects. These objects can include items such as cubes, wooden blocks, beads, rods, prisms, spoons, plates, apples, and oranges. Practitioners can obtain these objects from the environment according to the mathematical concepts and skills they intend to teach, as well as the types of concepts and operations, and use them in the instructional process. Especially for elementary school students and individuals with special needs, it is necessary to use these objects for concretization. However, concrete materials alone do not serve to make sense of mathematical concepts and skills. Students also need to reflect on and construct meaning from the concepts and skills (Sarama & Clements, 2009). Therefore, practitioners should not rely solely on manipulatives, treating them as if the manipulatives themselves are performing the instruction. Instructional adaptations should be made considering various factors such as students' individual differences, learning characteristics, and performance levels in mathematical concepts and skills. Consequently, manipulatives should be used as a means rather than an end.

As mentioned, virtual manipulatives are computer-based simulations of concrete manipulatives (Bouck & Flanagan, 2010). Thus, the study first addresses the definition of concrete manipulatives and their role in the process of imparting mathematical concepts and skills before discussing virtual manipulatives. Just like with concrete manipulatives, in a teaching process that will use virtual manipulatives, it is necessary to select a virtual manipulative platform or application appropriate to the concepts and skills to be taught. Unlike concrete manipulatives, which are obtained from the surrounding environment, virtual manipulatives are obtained from digital environments.

3.Virtual Manipulatives

Virtual manipulatives are defined as “an interactive, technology-enabled visual representation of a dynamic mathematical object, including all of the programmable features that allow it to be manipulated, that presents opportunities for constructing mathematical knowledge." (Moyer-Packenham & Bolyard, 2016). As the definition suggests, like concrete manipulatives, virtual manipulatives enable students to manipulate mathematical knowledge, i.e., mathematical concepts and skills. This allows students to actively participate in their learning processes (Satsangi & Bouck, 2015). Advanced technologies are used in the presentation of virtual manipulatives, which are delivered through technological tools such as tablets, computers, phones, and smart boards.

Manipulating virtual manipulatives does not simply mean displaying them on a tablet, computer, phone, or smart board screen. Moyer, Bolyard and Spikell (2002) explain that individuals often use the term virtual manipulative to refer to images created solely for representation on a screen, but the scope of virtual manipulatives is much broader. For example, in teaching the addition operation 2+3=? the teacher dragging two apples with their finger to a different point on the tablet screen and then bringing three more apples next to them to perform the addition operation exemplifies the manipulability of objects. In other words, objects need to be movable, transformable, addable, or augmentable by the practitioner or student. Therefore, virtual manipulatives are interactive digital visual representations of objects that provide opportunities to construct mathematical knowledge (Moyer et al., 2002). Figure 2 illustrates examples of virtual manipulatives, highlighting their differences from concrete manipulatives.

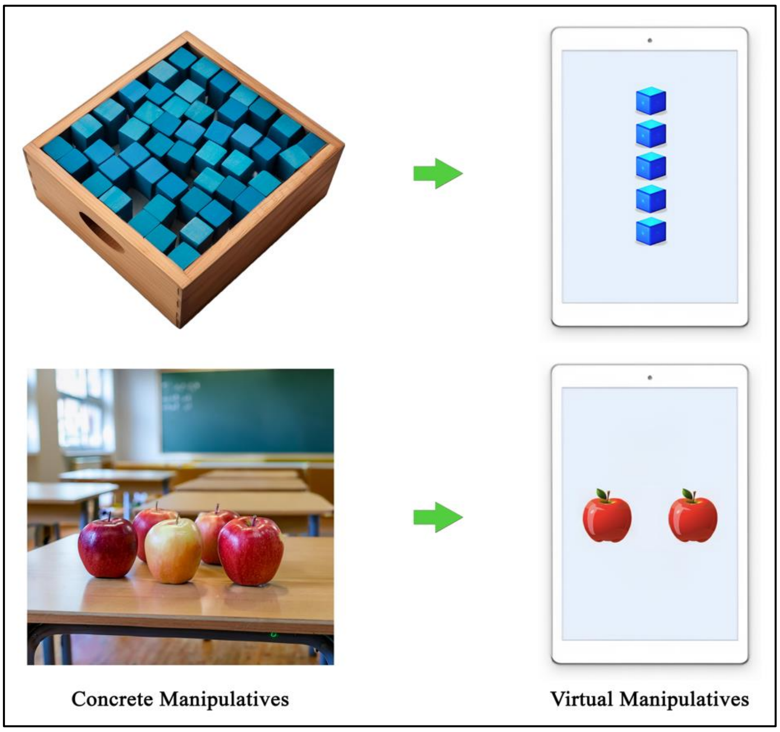

Figure 2. Concrete and virtual manipulatives*

As seen in Figure 3, virtual manipulative applications present an interactive process. They allow data input through computer hardware such as tablet-phone-smart board screens and a mouse, providing opportunities for practitioners and students to manipulate objects. Additionally, they contain elements that facilitate the construction of knowledge related to mathematical concepts and skills. In the later parts of the study, the virtual manipulative platforms introduced include applications that enable the teaching of mathematical concepts and skills.

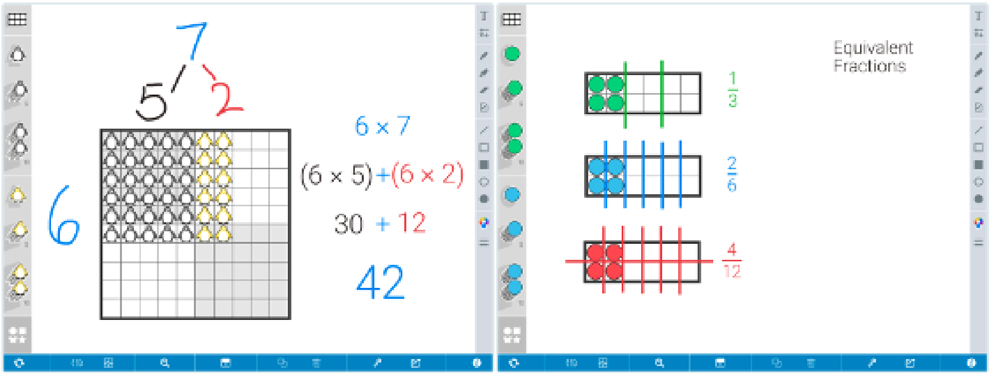

Figure 3. Virtual manipulative applications (Retrieved from https://www.mathlearningcenter.org/apps/number-frames on 02/06/2024).

Virtual manipulatives can be offered as application-based tools in addition to web-based platforms. These applications allow educators to easily adapt teaching processes through technology, providing opportunities for individual or group work with virtual versions of concrete manipulatives (Bouck & Flanagan, 2010). Virtual manipulatives come in various forms depending on their digital environment: single representation, multiple representation, tutorial, game, and simulation (Moyer-Packenham & Bolyard, 2016). Table 1 provides explanations of these digital environments. It is recommended to choose the appropriate environment based on students' learning characteristics and individual differences.

Table 1. *Digital environments for virtual manipulatives (Adapted from Moyer-Packenham & Bolyard, 2016)*

Environments	Explanations
Single Representation	It has an interactive visual representation of the mathematical object and contains no numerical or textual information.
Multiple Representation	It includes an interactive visual representation of the mathematical object along with numerical and sometimes textual information.
Tutorial	Similar to multiple representations, it includes numerical and textual information. In addition, it provides guidance and instructional support related to operations.
Game	It involves playing with visual representations of mathematical objects to achieve game objectives.
Simulation	It includes interactive visual and numerical representations of mathematical objects. Mathematical concepts can be simulated.

3.1. Virtual Manipulatives and Instructional Strategies

The use of virtual manipulatives alone can produce positive outcomes, but they can also be part of a broader implementation process (Bouck, Park, Cwiakala & Whorley, 2020). The relevant literature shows that virtual manipulatives are presented based on various instructional strategies. Among these strategies, the most common are the virtual-representational-abstract (e.g., Bouck, Long & Bae, 2023; Smith, 2023; Terzioğlu, 2020) and the virtual-abstract instructional strategies (e.g., Bouck, Park, Cwiakala & Whorley, 2020; Bouck, Park, Sprick, Shurr, Bassette & Whorley, 2017).

The concrete-representational-abstract instructional strategy, based on the use of concrete manipulatives, naturally involves the use of concrete manipulatives at the concrete stage. Based on the definition of the concrete-representational-abstract instructional strategy, a framework for the virtual-representational-abstract instructional strategy will be outlined. Indeed, in the literature, the virtual-representational-abstract instructional strategy is explained by starting from the concrete-representational-abstract instructional strategy (e.g., Bouck & Sprick, 2019; Root, Cox, Gilley & Wade, 2021).

The concrete-representational-abstract instructional strategy consists of three stages: concrete, representational, and abstract. In the concrete stage, students learn by using concrete manipulatives. In the representational stage, visual representations of concrete manipulatives are used. In the abstract stage, students work with abstract mathematical expressions such as numbers and operation symbols (Witzel, Riccomini & Schneider, 2008). In the virtual-representational-abstract instructional strategy, the representational and abstract stages are carried out in the same way as in the concrete-representational-abstract instructional strategy. However, in the concrete stage, virtual manipulatives are used instead of concrete manipulatives (Bouck, Bassette, Shurr, Park, Kerr & Whorley, 2017). Figure 4 illustrates the differences between these instructional strategies through visual representations.

According to Bouck and Sprick (2019), each stage in the virtual-representational-abstract instructional strategy is independent and distinct. In the virtual stage, representations are readily available on the screen. They can be rotated and viewed in three dimensions. In the representational stage, students create their own drawings of the representations. The abstract stage is entirely different from these two stages. Instead of the visuals and representations of objects, students use mathematical expressions and emphasize mathematical strategies. The process follows the steps of explicit instruction, including modeling, guided practice, and independent practice. Each stage is repeated in virtual, representational, and abstract formats.

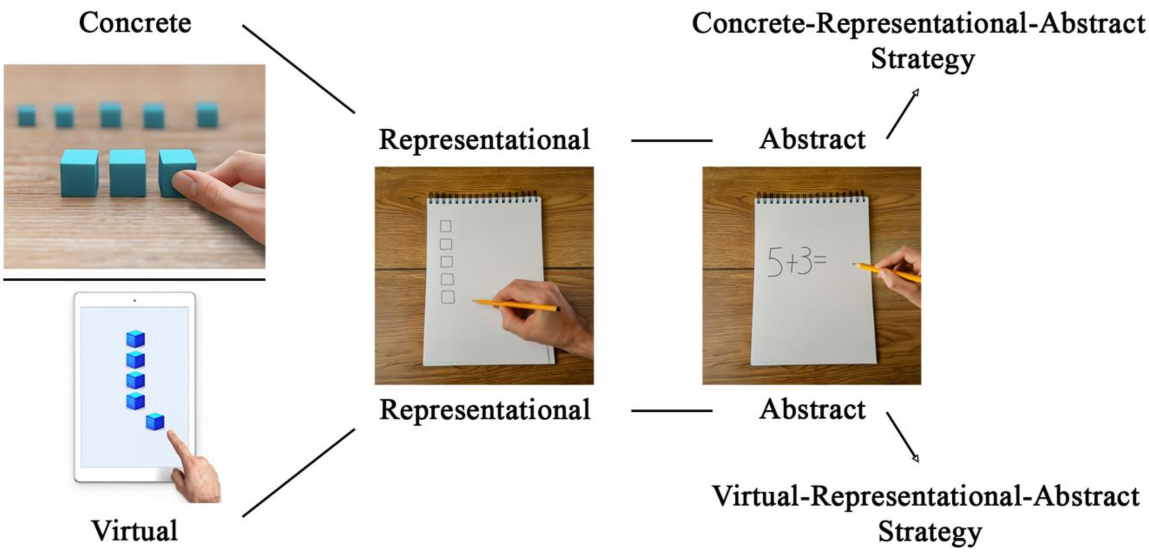

Figure 4. Concrete-representational-abstract and virtual-representational-abstract strategy*

The studies conducted based on the instructional sequence reported by Bouck and Sprick (2019) were identified through a literature review. In the studies conducted by Bouck et al. (2023) and Park, Bouck and Fisher (2021), they illustrated the process of conducting each stage within the explicit instruction framework with examples based on dependent variables. Based on these processes and perspectives, the instructional process based on the virtual-representational-abstract strategy is explained below.

The process of modeling, guided practice, and independent practice in the virtual stage: The practitioner uses a technological tool (e.g., computer, tablet, phone) at all steps of the virtual stage. The student also uses the same technology tool during guided and independent practice. The practitioner can introduce the platform they will use and state questions related to the mathematical concept or skill to be taught (e.g., “I will solve 3+4=?!”). Then, using virtual manipulatives created for this operation (see the virtual stage in Figure 4), the practitioner models the solution process. The practitioner continues to think aloud throughout the process. In guided practice, the practitioner asks the student to solve the operations. If the student performs the operation correctly, they provide reinforcement; if the student makes an error, they provide prompts. In independent practice, the practitioner asks the student to solve the operations independently without providing any prompts if the student makes an error. If the student reaches the expected performance level in the virtual stage, they move on to the representational stage.

The process of modeling, guided practice, and independent practice in the representational stage: In the representational stage, the practitioner informs the student that they will not use any technological tools. They then use paper and pencil to create drawings related to the operation and model the process (see the representational stage in Figure 4) (e.g., “I drew 3 lines here. I will draw 4 lines here. How many lines do I have? 1-2-3-4-5-6-7! I added 3 and 4”). The practitioner continues to think aloud throughout the process. In guided practice, the practitioner asks the student to solve the operation by drawing. If the student performs the operation correctly, they provide reinforcement; if the student makes an error, they provide prompts. In independent practice, the practitioner asks the student to solve the operations independently without providing any prompts if the student makes an error. If the student reaches the expected performance level in the representational stage, they move on to the abstract stage.

The process of modeling, guided practice, and independent practice in the abstract stage: The abstract stage represents the desired level of understanding for students regarding concepts and skills. In other words, it is the stage where formal mathematical expressions are used (see the abstract stage in Figure 4). Therefore, the practitioner models the solution of operations using numbers. They continue to think aloud throughout the process. In guided practice, the practitioner asks the student to solve the operation using numbers. If the student performs the operation correctly, they provide reinforcement; if the student makes an error, they provide prompts. In independent practice, the practitioner asks the student to solve the operations independently without providing any prompts if the student makes an error.

Another instructional strategy investigated in the literature for its effectiveness and reported with positive outcomes is the virtual-abstract instructional strategy. In the virtual-abstract instructional strategy, the representational stage is omitted. The instruction moves directly from the virtual stage to the abstract stage (Bouck, Park, Maher, Levy & Cwiakala, 2019). As seen in Figure 5, after students meet the criteria set in the virtual stage during the learning process with virtual manipulatives, they transition to mathematical expressions. The instructional process is conducted following the steps of modeling, guided practice, and independent practice in both the virtual and abstract stages, as described above.

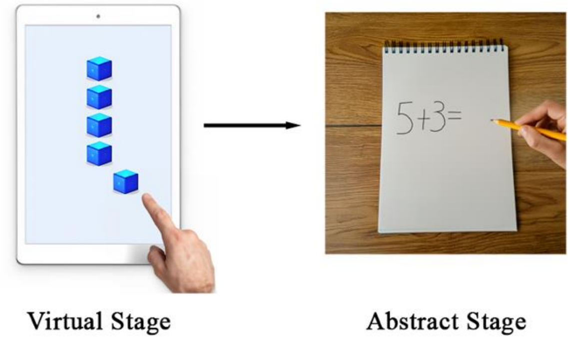

Figure 5. Virtual-abstract strategy*

There are several advantages to using virtual manipulatives. Satsangi and Miller (2017) state that virtual manipulatives are available at no or low cost, are time-efficient, can be easily shared among practitioners, and are accessible. They also mention that these tools can be adapted to various levels and can be easily assessed using rubrics. As stated, virtual manipulatives offer numerous advantages to practitioners before, during, and after instruction. Of course, the benefits they provide to students are continuously supported by literature within the framework of ongoing research. In this regard, Moyer-Packenham and Westenskow (2013) conducted a meta-analysis study examining the effectiveness of virtual manipulatives. They state that, based on the findings, virtual manipulatives focus attention on mathematical objects and processes, enhance creativity, simultaneously connect representations to each other and to actions, provide precise representations, and motivate students to continue engaging in mathematical activities. Thus, the research indicates that virtual manipulatives have positive effects both in the teaching process and in students' learning experiences. However, there are also some disadvantages associated with virtual manipulatives. Their use requires various technological tools, which can be challenging to acquire; additionally, students with limited fine motor skills may have difficulty using these tools (Satsangi & Miller, 2017).

3.2. Virtual Manipulative Platforms

There are various web-based platforms and applications that practitioners can use to teach mathematical concepts and skills to students with special needs. Although these platforms and applications are primarily in English due to their developers, it is believed that they are easily accessible for practitioners whose first language is not English. In this section of the research, web-based and application-based virtual manipulative platforms that can be used in teaching processes will be discussed. Naturally, it will not be possible to cover all platforms due to the continuous increase in such platforms. Therefore, the most commonly encountered platforms have been highlighted based on the scans and reviewed studies.

3.2.1. Polypad (https://polypad.amplify.com)

Polypad is a web-based mathematics platform developed by Amplify. It includes a variety of virtual manipulatives that can be used to teach mathematical concepts and skills. Some of these manipulatives include fraction bars, 3D digital objects, and balances. Polypad is free to use and can be accessed from any device with internet connectivity.

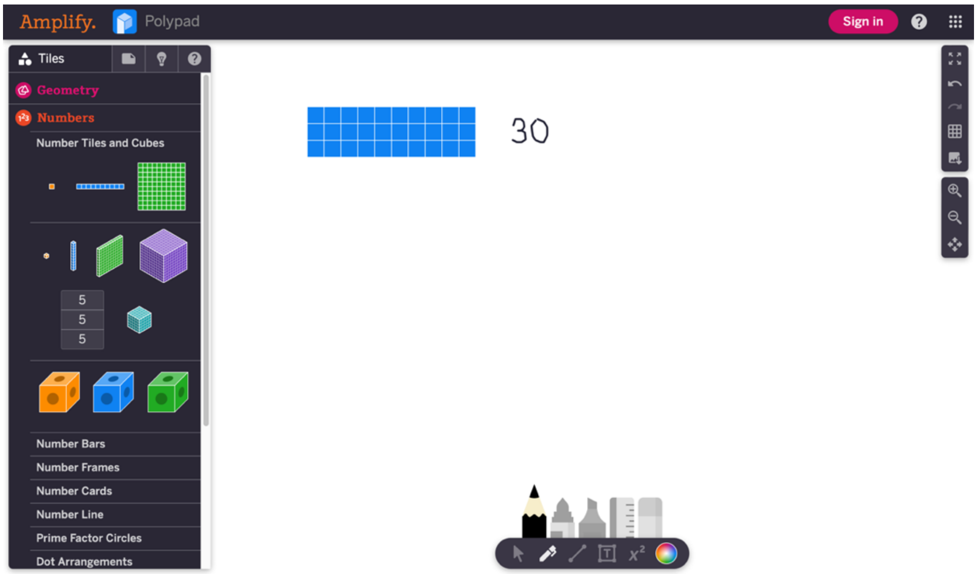

Figure 6. Amplify-Polypad virtual manipulative platform (Created using the link https://polypad.amplify.com/p#number-tiles on 24.06.2024.)

3.2.2. Brainingcamp (https://www.brainingcamp.com)

Brainingcamp is another web-based virtual manipulative platform that can be used to teach mathematical concepts and skills. It includes a variety of virtual manipulatives such as algebra tiles, base ten blocks, clocks, and fraction circles. While Brainingcamp is a paid platform, it offers a 30-day free trial. It allows teachers and students to conduct interactive learning processes. Additionally, Brainingcamp features blog posts that provide a wealth of information about using virtual manipulatives. Furthermore, Brainingcamp offers a virtual manipulative app that is available for use on iPads.

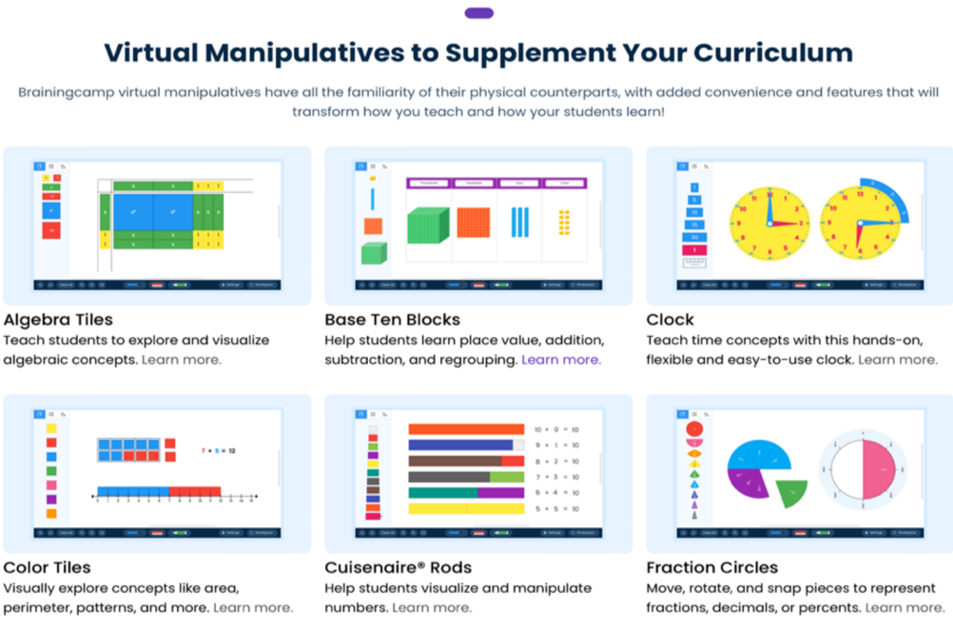

Figure 7. Brainingcamp virtual manipulative platform (Created using the link https://www.brainingcamp.com/manipulatives on 24.06.2024)

3.2.3. Didax (https://www.didax.com/math/virtual-manipulatives.html)

Another virtual manipulative platform that can be used to teach mathematical concepts and skills is the virtual manipulatives developed by Didax. This platform includes cubes, number lines, colored tiles, dice, a geometric board, and many other virtual manipulatives. It is observed that the use of Didax's virtual manipulatives is free of charge.

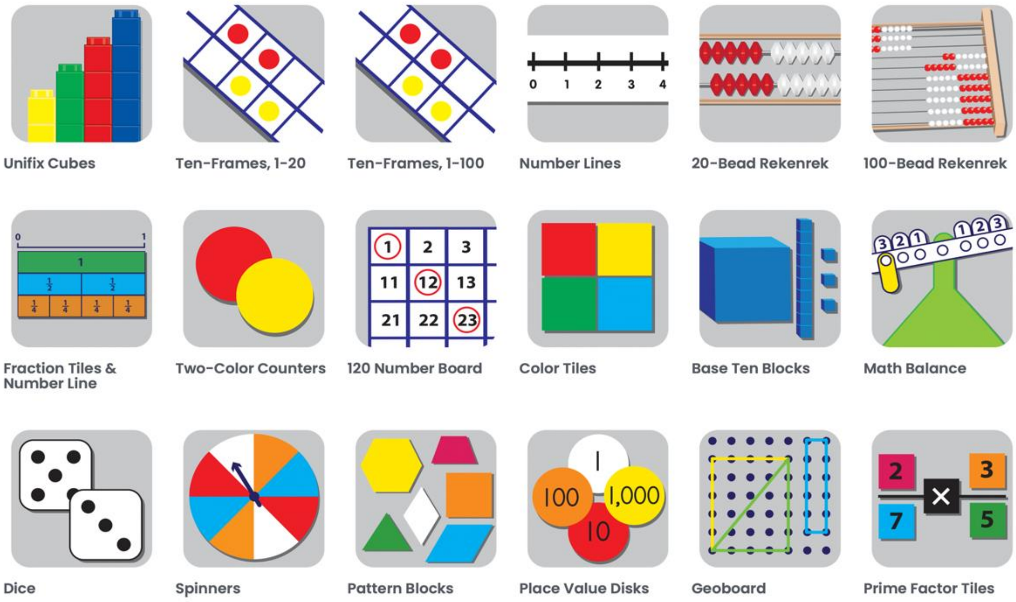

Figure 8. Didax virtual manipulative platform (Created using the link https://www.didax.com/math/virtual-manipulatives.html on 24.06.2024.)

3.2.4. ToyTheater (https://toytheater.com)

ToyTheater includes many virtual manipulatives, including games, to teach mathematical concepts and skills. The platform provides numerous virtual manipulatives that allow students to learn while having fun. In addition to tools for teaching mathematical concepts and skills, it also offers resources for other developmental areas.

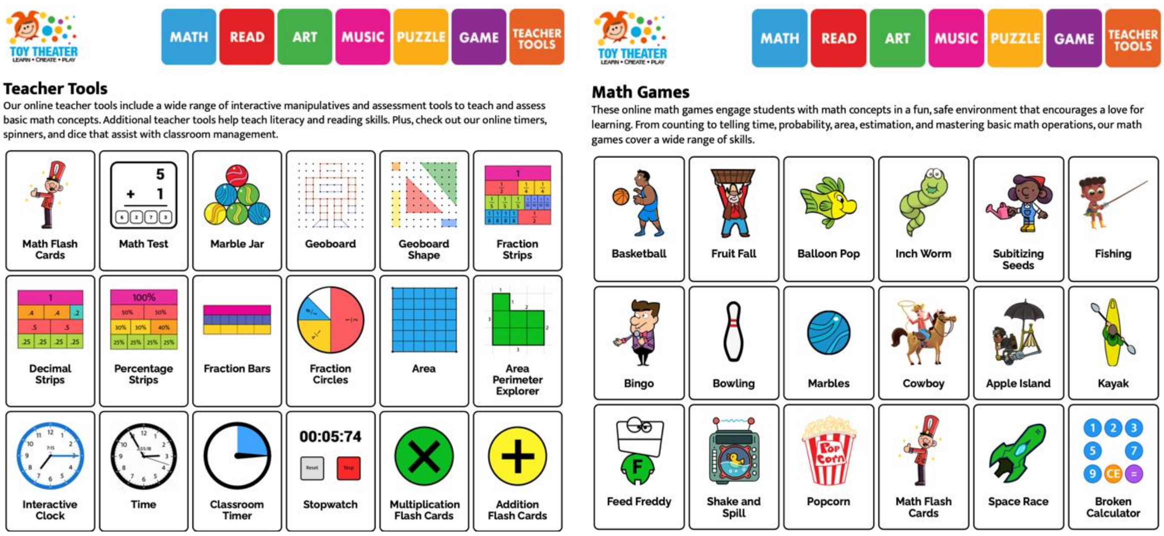

Figure 9. ToyTheater platform (Created using the links https://toytheater.com/category/math-games/ and https://toytheater.com/category/teacher-tools/ on 24.06.2024.)

4. Studies in the Literature

Students with special needs need to learn mathematical skills, one of the functional academic skills, to live independently and succeed in school. The virtual-representational-abstract instructional strategy can be said to be one of the effective teaching strategies for teaching mathematical concepts and skills to students with special needs. It has been found that there are many studies in the literature that examine the effectiveness of virtual manipulatives, the virtual-representational-abstract instructional strategy, and the virtual-abstract instructional strategy in teaching mathematical concepts and skills to students with special needs. There are also studies comparing the effectiveness of concrete and virtual manipulatives. The dependent variables in the studies include skills such as addition, subtraction, multiplication, and division, operations with fractions, place value, and problem-solving. The participants in these studies were typically students with learning disabilities, intellectual disabilities, and autism spectrum disorder. Some studies also included students with special needs from different diagnostic groups. This is specifically noted in the studies described below.

In a study by Bouck, Park, Shurr, Basette, and Whorley (2018), the effectiveness of the virtual-representational-abstract instructional strategy was examined in teaching place value, addition, subtraction, and multiplication skills to two middle school students with intellectual disabilities. The results of the study, conducted using a multiple probe design across behaviors, show that this strategy is effective in teaching these skills. However, the researchers noted that additional strategies might be needed for the maintenance of these skills. In a study by Terzioğlu (2020), the effectiveness of the virtual-representational-abstract instructional strategy was examined in teaching the four basic operations to seven students with intellectual disabilities. The study also included generalization sessions and collected social validity data. The results, obtained using a multiple probe design across participants, indicate that the virtual-representational-abstract instructional strategy is effective, and students could maintain and generalize their gains across different environments and people. Social validity data also showed that teachers had positive views about the implementation process.

In the study by Prabavathy, Sivaranjani, and Alex (2023), the effectiveness of virtual manipulatives was examined in teaching place value to eight students with intellectual disabilities. The results of the study, conducted using a single-group experimental design, show that virtual manipulatives can be effective in teaching place value to students with intellectual disabilities. The researchers also suggested that adopting such learning environments could increase the passion for learning and make it more enjoyable. In the study by Park et al., (2021), the effectiveness of the virtual-representational-abstract instructional strategy was examined in teaching multiplication skills to students with special needs. The study involved one student with autism spectrum disorder and two students with learning disabilities, using a multiple probe design across participants. The results indicate that the virtual-representational-abstract instructional strategy is effective, and students were able to maintain their gains. Additionally, both teachers and students had positive views about the strategy.

There are many other studies in the literature that examine the effectiveness of virtual manipulatives. For example, studies have investigated the effectiveness of virtual manipulatives in teaching area and perimeter concepts (Satsangi & Bouck, 2015), equation-solving skills (Satsangi, Bouck, Taber-Doughty, Bofferding & Roberts, 2016; Satsangi, Hammer & Evmenova, 2018; Satsangi, Hammer & Hogan, 2018), algebra (Bouck, Park, Satsangi, Cwiakala & Levy, 2019), division with remainders (Bouck, Park & Stenzel, 2020), and addition operations (Hammons, 2019). There are also studies examining the effectiveness of the virtual-representational-abstract instructional strategy in teaching subtraction skills

(Park, 2019), multiplicative problem-solving skills (Root et al., 2021), and fractions (Bouck, Bassette et al., 2017), all of which show effective results.

Based on these studies, it can be said that virtual manipulatives and instructional strategies can have positive effects on teaching various mathematical concepts and skills to students with special needs. The use of virtual manipulatives can motivate students and increase their interest in learning (Moyer-Packenham & Westenskow, 2013; Prabavathy et al., 2023). It can be suggested that more research needs to be conducted in relevant contexts.

5. Conclusions

One of the skill areas that students with special needs need to exhibit to live independently and succeed in school is mathematics. The use of emerging technologies in the education of students with special needs is becoming increasingly common, and its impacts are also seen in the teaching processes of mathematical concepts and skills. Virtual manipulatives serve to concretize the abstract concepts and skills in mathematics, yielding positive results in mathematics education. It is believed that practitioners can achieve positive outcomes by using virtual manipulatives in the teaching of mathematical concepts and skills.

In the literature, it is observed that virtual manipulatives can be used to teach many mathematical concepts and skills such as addition, subtraction, multiplication, and division skills, problem-solving skills, area and perimeter, and fractions (e.g., Satsangi & Bouck, 2015; Hammons, 2019; Bouck et al., 2018; Bouck, Bassette et al., 2017; Root et al., 2020). Virtual manipulatives can be used directly in the application processes or within the framework of instructional strategies such as virtual-representational-abstract or virtual-abstract. Practitioners need to make decisions about instructional strategies and platform preferences based on the characteristics of their students. They can also determine the technological tools they will use, such as computers, tablets, or phones, within this context. Indeed, an application carried out using a computer will differ from one using a tablet or phone due to the type of data entry (e.g., using a keyboard and mouse). For example, the easy portability, small size, and personalization options of devices like tablets (Subakan & Koc, 2019) could be reasons for a practitioner to prefer using a tablet. Similarly, if a student prefers working with a computer over these devices, it could be another reason for the practitioner's preference.

It is believed that ongoing developments and research in the context of technological tools and software will also impact virtual manipulative platforms. Accordingly, it is considered necessary for practitioners to continuously follow these developments and stay updated. Practitioners will be able to incorporate advancements into their instructional processes appropriately by staying current in the technological age, while also considering individual differences and learning characteristics of their students when planning their teaching processes.

In this study, the role of virtual manipulatives in teaching mathematical concepts and skills to students with special needs has been examined. Instructional strategies that can be followed in the application processes, platforms that can be used, and several studies on the effectiveness of virtual manipulatives have been summarized. Undoubtedly, technology will continue to advance. In line with these developments, research on virtual manipulatives should continue, and their use in teaching mathematical concepts and skills to students with special needs should become more widespread.

6. Ethics approval

Since this is a review study, there is no need for the ethics committee approval.

7. Conflict of interest

The author declares that there is no conflict of interest.

8. Funding

No funding was received for this study.

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*Images were created using Adobe Firefly (https://firefly.adobe.com) and Adobe Photoshop 2024.