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EXAMINATION OF PRE-SERVICE CLASSROOM TEACHERS' KNOWLEDGE ON THE FOUR OPERATIONS

(Research Article)

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Abstract

The aim of this study was to examine pre-service classroom teachers' knowledge on four operations. In this context, Algorithm Test (AT), Modelling Test (MT) and Additional Strategy Test (AST) developed by the researcher were used to examine the algorithm, modeling and additional strategy knowledge of the pre-service teachers. Thus, the pre-service teachers were expected to use different types of knowledge simultaneously. The solutions of the pre-service teachers were first classified as correct/incorrect, and then the solution stages were examined in detail. 110 pre-service classroom teachers at a state university in the 2021-2022 academic year participated in the study. Explanatory mixed method, in which quantitative and qualitative methods are used together, was adopted in the study. The explanatory design is known as a method in which quantitative data are supported by qualitative data. The quantitative data were collected using operation tests consisting 4 open-ended questions about four operations, and the qualitative data were collected through semi-structured interview technique. The results showed that the success order of the participants in all test types was addition, subtraction, multiplication and division, respectively. The success rate of the participants in AT and MT was above 50% while the success rate in the division in AST was below 50%. It was also found that the participants used seven different strategies and preferred to use only standard models in modelling.

Keywords: four operations, algorithm, modelling, additional strategies, prospective teachers.

1. Introduction

One of the main goals of the Primary School Mathematics Curriculum (MEB, 2019) is to develop four operations skills in natural numbers in students. Now, it is a well-known fact that a solid knowledge of four operations is required to be successful in mathematics (National Council of Teachers of Mathematics [NCTM], 2000). Learning the four operations requires knowledge of number, positional notation, relations between operations, mental mathematics and association with models (Bana & Korbosky, 1995; Shuard, 1986).

Existing studies have revealed that children construct mathematics based on the information presented to them and the experiences they have (Cobb, 2000). Mathematics educators believe that students acquire mathematical skills, including four operation skills, under the guidance of their teachers (Gray, 2004; Owens, 2006). Thus, teachers should discover new ideas that will help students understand mathematics and be able to apply them (Beaudine, 2022). Fidan and Erden (1994) argues the most basic task of the teacher is to make the students understand the topic through various teaching techniques. In this sense, it can be said that most of the difficulties experienced in teaching mathematics derive from the teacher' way of instruction and therefore teacher education (Karaçay, 1985; Soylu, 2009). The way a particular subject is interpreted and conveyed to students is related to the teacher's pedagogical content knowledge. Shulman (1986; p.9) defines pedagogical content knowledge as "the knowledge required to make the content understandable for others and to represent it in different ways". Therefore, classroom teachers should be able to diversify and explain the content in mathematics lessons in accordance with the level of children. Sherin (2002) puts forward that teachers should be able to express mathematical ideas in more than one way. According to Lee (2010), high-quality and expert teachers not only have in-depth knowledge on the topic, but also know how to convey it to the students. In order to enable students to come up with more than one solution to problems, teachers should solve problems and operations in more than one way (Silver et al., 2007). In many studies, pedagogical content knowledge is regarded as an essential part of being an effective teacher (Leinhardt, 1986; Graeber, 1999; Stewart, 2008).

The Primary School Mathematics Curriculum (MEB, 2019) aims to make children do basic operations on addition and subtraction in the 1st grade and multiplication and division in the 2nd grade. The curriculum requires teaching the four operations not only with algorithms, but also using mathematical models and additional strategies. Particular emphasis is placed on objects that students use in real life, standard and non-standard models. According to the National Council of Teachers of Mathematics (NCTM, 2000), primary school students can do addition, subtraction, multiplication and division in numerous ways. Studies in the literature indicates that content knowledge and pedagogical content knowledge affect the teachers' practices in mathematics and student learning (Baumert et al. 2010). Similarly, there are studies showing that the quality of learning opportunities provided by teachers influence student learning and motivation (Hattie, 2009). Hill et al. (2005) reported that there was a positive relationship between classroom teachers' teaching styles and student achievements. Van De Walle (2019) proposes that the four operations should be taught in three ways: algorithm, modeling and student-invented strategies.

In-service and pre-service classroom teachers' pedagogical content knowledge of mathematics has been regarded as a basis for effective mathematics learning for many years. Turnuklu and Yeşildere İmre (2007) stated that although knowledge on mathematics is essential it is not sufficient for teaching mathematics. Lee (2005) and Lee et al. (2003) stated that pre-school teachers who had higher scores pedagogical content knowledge in mathematics provided education of higher quality in their classrooms and their students were more successful. In many studies (Aksu, 2008; Arslan & Özpınar, 2008; Bahşi & Güreş, 2021; Güner, 2013; Şahin, 2013) it was reported that pre-service teachers had professional qualifications and competencies in the field of mathematics. In contrast, Soylu (2009) found that pre-service teachers did not regard themselves sufficient in terms methods and techniques regarding the constructivist approach. In addition, Hoşşirin Elmas (2010) expressed that classroom teachers who did not take mathematics teaching course in teacher education had high levels of anxiety which was due to their lack of mathematical knowledge and content knowledge. Similarly, a number of studies (Arseven et al., 2015; Çağrığan Gülten, 2011; Çekirdekçi, 2021; Hacıömeroğlu, 2011) examined the attitudes of pre-service teachers towards teaching how to learn mathematics and reported that the attitudes of classroom teachers were lower than those of mathematics teachers, which should be investigated. Examining pre-service classroom teachers' mathematics knowledge for teaching, Hacıömeroğlu (2013), stated that pre-service teachers' specialized content knowledge was weak and their pedagogical content knowledge for addition and subtraction was based on their tendency to use solution approaches that they named as short and practical. Çelikten et al. (2005) argues that one of the reasons for such a situation is that the pre-service teachers are not prioritized in practicum schools. Aslıoğlu (2006), on the other hand, attributes such a situation to the limited opportunities for pre-service teachers to apply the knowledge and techniques they acquired during their undergraduate education.

The four basic operations are the foundation of mathematics, and therefore, individuals who fail to learn them cannot be expected to be successful in other mathematics topics and even in life (Brandt et al., 2016). The Primary School Mathematics Curriculum (MEB, 2019) includes achievements aboıt four operations from the 1st grade to the 4th grade. The curriculum recommends the use of algorithms, modeling and additional strategies in teaching the four operations. The algorithm consists of a set of rules such as starting addition, subtraction, multiplication from the right, and division from the left. The students should be encouraged to discover these rules through several models and strategies (Van De Walle et al., 2014). Solving the operations with algorithms alone will be limited to memorizing a meaningless set of rules (Swan & Marshall, 2010). Such an instruction also prevents students from understanding operations based on positional notation (Eitel et al., 2013). In this sense, Baki (2013) stated that pre-service classroom teachers mostly did division operations correctly, but their instructional explanations on positional notation were insufficient. Hence, four operations problems should be solved not only operationally with the algorithm, but also through additional strategies using the relationships between numbers (Van De Walle et al., 2014). According to Beaudline (2022), teachers should adopt strategies they consider effective in the classroom, present new strategies to students, and model each strategy. Similarly, Senemoğlu (2005) argues that strategies increase storage in short-term memory and strengthen comprehension and retention. According to Montague (1998), students' failure in mathematics result from their ignorance of strategy. Many previous studies have revealed that teachers are not sufficient and do not prefer to use additional strategies in Mathematics lessons (Bahar, 2019; Bozkurt & Yavaşça, 2021; Gürbüz & Güder, 2016; Şengül & Gülbağcı Dede, 2014; Yeşildere & Akkoç, 2010).

Teachers can attract students' attention and facilitate their learning through the use of a number of models in their classrooms. Therefore, the use of models in school plays a key role in education (Rosli et al., 2015). According to NCTM (2000), teachers should be able to decide on the most appropriate method to enable students to learn mathematical concepts successfully. Similarly, the teachers are required to be able to use models adaptably and to present their mathematical ideas through models (Eitel et al., 2013). Models refers to the concrete system of abstract mathematical ideas (Van De Walle et al., 2014), and the ability to model bring out the relationship between mathematical concepts and operations (Swan & Marshall, 2010). Through the models, students' perceptions on many subjects such as numbers, four operations, and positional notation can be improved (Barnett-Clarke et al., 2010). In this context, previous studies examined the modeling skills of pre-service teachers and found that they had difficulties in modeling (Aksu & Konyalıoğlu, 2015; Bayazit et al., 2011; Kertil, 2008; Dede & Yılmaz, 2013). In this sense, Korkmaz (2010) investigated the perspectives of pre-service classroom and mathematics teachers on modelling and found that there was no significant difference. However, pre-service teachers expressed that they considered modeling complex, but with modeling, they realized the importance of mathematics in daily life. In addition, Duran et al. (2016) reported that pre-service mathematics teachers had difficulties in modeling and that their proposed models mostly did not comply with the logic of the problem. In contrast, Suh et al. (2017) found that the students of teachers who employed modeling in their classrooms understood and structured mathematical ideas more easily. Similarly, pre-service teachers in Saka and Çelik (2018) stated that the technology facilitated the mathematical modeling process. Tekin Sitrava et al. (2020) examined pre-service classroom teachers' content knowledge on the meaning of division and found that more than half of the participants had insufficient knowledge of modeling division. Alsina and Salgado (2021) put forward that teachers contribute greatly to the development of students' modeling skills and that teachers who use modeling in their classrooms from the early years are more successful in concretizing and modeling mathematics.

Based on the brief review above, the aim of this study was to examine the pre-service classroom teachers' knowledge on four operations. Algorithm, modeling and additional strategy knowledge and difficulties experienced by pre-service teachers were discussed in detail. Therefore, answers to the following problems were sought:

1. What is pre-service classroom teachers' knowledge level in the questions about the four operations?

2. What is pre-service classroom teachers' algorithm knowledge in the questions about the four operations?

3. What is pre-service classroom teachers' modelling knowledge in the questions about the four operations?

4. What is pre-service classroom teachers' knowledge of producing additional strategies in the questions about the four operations?

5. Is there a significant difference between the developed tests?

6. What are the strategies and models employed by the pre-service classroom teachers in the questions about the four operations??

7. What are the difficulties experienced by the pre-service classroom teachers in algorithm, modeling and using additional strategies in the questions about the four operations?

2. Method

In this research, a mixed method combining quantitative and qualitative techniques was employed. In the first stage, quantitative data were collected and analyzed, and in the second stage, qualitative data were collected to obtain in-depth information about quantitative data and to support the findings. In this sense, explanatory sequential design, one of the mixed research methods, was used (Uygun, 2012). In this design, first quantitative data is collected and analyzed, and then qualitative data are used for the situations that cannot be explained with quantitative data (Creswell & Plano-Clark, 2007; Fraenkel et al., 2012). In the study, the achievement of the pre-service teachers in the four operations was examined with quantitative methods, and how they solved the four operations questions was examined with qualitative methods. Therefore, quantitative results were interpreted based on qualitative results.

2.1. Participants

Criterion sampling method, a purposive sampling method, was used in the sample selection (Fraenkel, Wallen and Huy, 2011). The inclusion criterion in the study was to study in the Department of Primary Education and to have taken Mathematics Teaching I and II courses. Since no generalization would be made in the qualitative part of the study, the universe and sample selection was not made and thus all pre-service teachers were invited to the interviews and 110 pre-service teachers agreed to participate in the interviews voluntarily.

2.2. Data Collection Tools

2.2.1. Quantitative Data Collection Tools

A number of tests were developed by the researcher in order to collect data. These data collection tools are explained below.

The Algorithm Test (AT), Modeling Test (MT) and Additional Strategies Test (AST) were developed by the researcher in order to examine the participants'' knowledge level about four operations in the Primary School 4th Grade Mathematics Curriculum. In each test, there were 4 open-ended questions for addition, subtraction, multiplication and division. Pre-service teachers were asked to answer each question in the tests. The same addition, subtraction, multiplication and division operations were included in all three tests However, it was aimed to examine the participants' algorithms knowledge in AT, modeling knowledge in MT and additional strategy knowledge in AST. Equal points were given to each question in the tests. The data obtained from the tests were evaluated as true or false. 1 point was given for each correct answer and 0 points for each incorrect answer. Therefore, the highest score that can be obtained in each test was 4 and the lowest score was 0.

Q1 and Q2 in AT were as follows:

$$\begin{array}{r} 109 \\ \underline{+199} \end{array} \qquad \begin{array}{r} 109 \\ \underline{-99} \end{array}$$

Please solve the questions above with the algorithm and explain your solution.

Q4 in AT is presented below:

$$1003 \,\big|\, \underline{5}$$

Please solve the question by modeling and explain your solution.

Q3 in AST was as follows:

$$\begin{array}{r} 109 \\ \underline{x23} \end{array}$$

Please solve the question by using additional strategies and explain your solution.

In order to ensure the content validity of the tests, the final versions of the tests were developed by obtaining expert opinions from 4 experts on mathematics education in primary school. The tests were applied to 42 pre-service classroom teachers in a state university as a pilot study. After piloting, the KR-20 reliability coefficient was calculated to examine the reliability of the tests. As a result, KR-20 value was found to be.78 for AT, .72 for MT and .78 for AST.

2.2.2. Qualitative Data Collection Tools

In the qualitative part of the study, the answers of the participants were examined. The models and additional strategies that the participants employed more in the four operations questions and the challenges they experienced were investigated. In this context, interviews, which were audio-recorded, were conducted with each participant. Participants were given a sheet of paper to write down their solution methods.

2.3. Data Analysis

After controlling the data collected from the tests, the scores of each participant were digitalized and statistical analyzes were performed based on research questions using SPSS. Relationships between tests were examined through the Pearson's Product-Moment Correlation Coefficient. The coefficients calculated between 0.3 and 0.7 were considered as moderate, coefficients larger than this value as strong, and coefficients smaller than this value as weak (Köklü & Büyüköztürk, 2000:107).

Both content analysis and descriptive analysis were used to analyze the qualitative data. The purpose of content analysis is to categorize related data on the basis of particular concepts and themes and to classify and interpret them in a comprehensible way (Yıldırım & Şimşek, 2019). In this sense, content analysis was used to examine the misconceptions experienced by the participants in AT. The data obtained in the descriptive analysis technique were classified and interpreted in line with the previously determined categories (Yıldırım & Şimşek, 2019). In order to increase the reliability of the study, the answers of the participants were examined by the researcher and an expert, and the items with "agreement" and "disagreement" were identified. The following formula was used for the reliability of the study (Miles & Huberman, 1994).

Reliability = [(The number agreements) / (The number of agreements) + (The number of disagreements)] x 100

In order to for a study to be reliable, a reliability value of at least 70% is required (Yıldırım & Şimşek, 2019). In this study, the reliability value was found to be 92%, indicating that the study was reliable..

3. Results

In this section, answers to the research questions are presented. The quantitative and qualitative findings were presented, respectively.

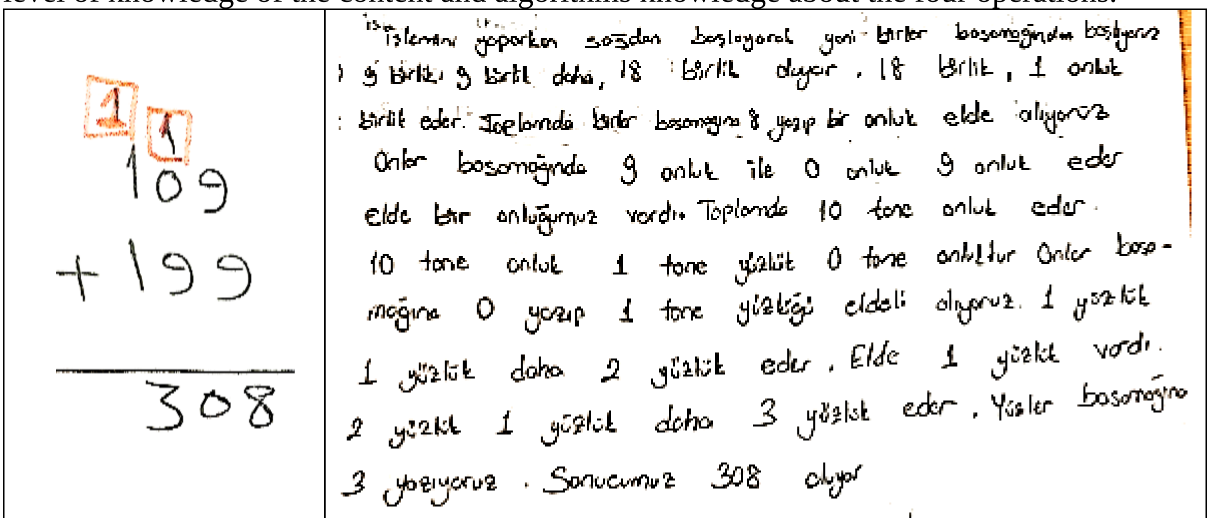
Findings on AT

In this study, the aim of which was to investigate the pre-service teachers' knowledge about four operations, an Algorithms Test (AT) developed by the researcher was applied to the participants. In the test, the participants were asked to solve problems about four operations using algorithm. The distribution of the participants who answered the questions correctly and incorrectly is shown in Table 1.

Table 1. *Distribution of answers in AT*

Operation Type	Correct		Incorrect		No Answer	
	f	%	f	%	f	%
Addition	105	95	5	5	-	-
Subtraction	102	93	8	7	-	-
Multiplication	102	93	8	7	-	-
Division	100	91	10	9	-	-

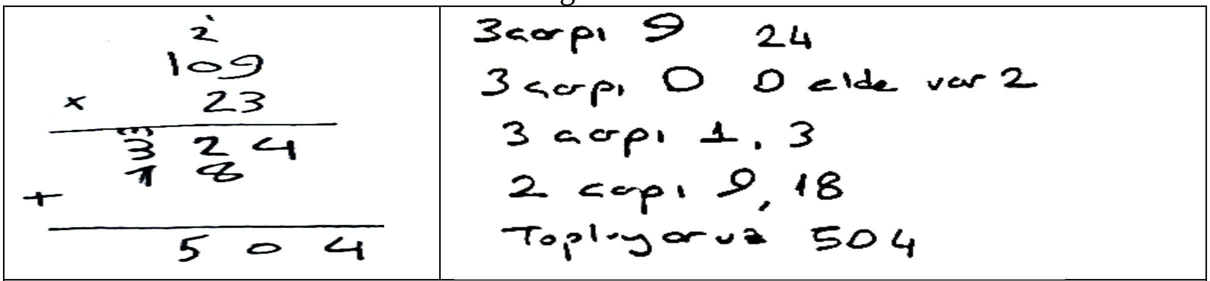
Table 1 showed that the success rate of the participants in the questions regarding the four operations in the AT was over 90%. Accordingly, it can be said that the participants had a high level of knowledge of the content and algorithms knowledge about the four operations.



[A1]

Let's start with the ones digit from the right. 9 units plus 9 units equals 18 units. 18 units equals to 1 ten and 8 units. In sum part, we write 8 in units and get a ten. In the tens place, 9 tens plus 0 tens equals 9 tens. We had a ten before. It makes 10 tens in total. 10 tens make 1 hundreds and 0 tens. We write 0 in the tens digit and get 1 hundreds. 1 hundred plus 1 hundred equals 2 hundred. We had a hundred in hand. 2 hundred plus 1 hundred equals 3 hundred. We write 3 in the hundreds place. As a result, we get 308.

Figure 1. The correct answer of a participants who solved the problem "109 + 99=?" with algorithms



3 times 9, 24. 3 times 0, 0 we have 2. 3 times 1, 3. 2 times 9, 18. Let's add we get 504.

Figure 2. The incorrect answer of a participants who solved the problem "109 x 23" with algorithms

As seen in Figure 1, the participant who answered the question about addition correctly made a correct instructional explanation by providing a justification based on the positional notation. All of the participants, whose answer was accepted correct, provided an answer similarly to that of the participant in Figure 1. On the other hand, the participant in Figure 2 provided an incorrect solution in terms of both the operation and the instructional explanation. The participants stated that 9x3 is 24 and did not consider 2 in the tens digit of the number 23. The answers of the participants who answered in this manner were considered incorrect/insufficient, indicating that their content knowledge for doing operations with the algorithm was weak.

Findings on MT

MT was administered to the participants in order to examine the participants' modeling knowledge in the four operations. Participants were asked to solve the questions in MT using standard and non-standard models. Table 2 shows the distribution of the participants' answers.

Table 2. *Distribution of answers in MT*

Operation Type	Correct		Incorrect		No Answer	
	f	%	f	%	f	%
Addition	101	92	2	1	8	7
Subtraction	91	83	9	8	10	9
Multiplication	86	79	2	1	22	20
Division	67	61	3	3	40	36

Table 2 revealed that the success rate of the participants in all questions was above 50%. Participants were most successful in addition (92%) and least in division (61%) in the modeling test. Based on this finding, it can be said that more than half of the participants had a high level of content and modeling knowledge in four operations.

Findings on AST

AST was applied to the participants in order to examine their additional strategy knowledge. In this sense, a number of additional strategies such as grouping, rounding, combining, scattering etc. were used and the success rates for the four operations questions were determined. Table 3 presents the distribution of the participants' answers.

Table 3. *Distribution of answers in AST*

Operation Type	Correct		Incorrect		No Answer	
	f	**%**	**f**	**%**	**f**	**%**
Addition	97	88	7	6	6	6
Subtraction	90	82	9	7	12	11
Multiplication	73	66	5	5	32	29
Division	43	39	7	6	60	55

As seen in Table 3, the AST success rate of the participants was more than 50%, except for the division. As with MT, participants were the most successful in addition (88%) and the least in division (39%). These findings indicated that more than half of the participants had a high level of additional strategy knowledge in addition, subtraction and multiplication. On the other hand, the participants' content and additional strategy generation in division was weak. All types of transactions show that empty answers are more than incorrect answers. This may be said to be due to poor field information and additional strategy generation information for the candidates of the teacher.

The results of AT, MT and AST revealed that the order of operations in which the participants were successful did not change, which was as follows: addition, subtraction, multiplication and division. In the AT and MT, the success rate for all operation types was above 50%, while the success rate of the division operation was below 50% in the AST. Based on these results, it can be put forward that the participants mostly had more difficulties in modeling and producing additional strategies

Findings on the Relationship between AT, MT and AST

Table 4 shows the examination whether there was a significant difference between the participants' scores in AT, MT and AST.

Table 4. *The examination of the relationship between AT, MT and AST*

Source	DF	SD	MS	F	p*
Groups	2	1428.5	714.25	2.7595	0.1162
Error	9	2329.4999	258.8333		
Total	11	3757.9999	341.6364		

It was found that there was no significant difference between the tests (p*>0.05).

Difficulties experienced by the participants in AT

Although the participants were largely successful in solving the questions about the four operations, they had difficulties in providing instructional explanation to them. For example, as seen in Picture 3, a participant explained the addition process as "9 plus 9 equal 18" instead of "9 units plus 9 units equal 18 units". In such an explanation, it is not understood which digit is units, tens and hundreds, and how many tens and hundreds transferred. The same problem existed in subtraction. Although 1 hundred, that is, 10 tens, was transferred from the hundreds digit, the participant stated that "We transfer 1 ten".

9, 9 daha 18, 8'i yazıyorum 1'i elde tutuyorum. 0, 9 daha 9, elde var 1 etti 10, onun 0'ı. 1 elde yine. 1, 1 daha 2, 1 elde etti 3 Sonuç 308.	9 plus 9 is 18. I write 8 and get 1 in hand. 0 plus 9 is 9. I had 1. It equals 10. 0 of 10. I get 1 in hand again. 1 plus 1 is 2. I had 1. It equals 3. The result is 308

9'dan 9 çıktı 0, 0'dan 9 çıkmaz 1onluk aldık 10'dan 9 çıkarsa 1 Sonuç 10	9 minus 9 is 0. We cannot subtract 9 from 0. We borrow 1 tens. 10 minus 9 is 1. The result is 10.

Figure 3. Examples of the answers of the participants who correctly solved ”109+99=?” ve”109–99=?” using the algorithm, but provided incorrect/insufficient explanations

Similar to addition and subtraction, in multiplication and division problems, the participants had difficulties in providing instructional explanation to the operations. It can be said that the participants had difficulty in explaining the operations because they did not consider multiplication and division as "number of groups x number of objects = total number of objects".

3. 109 x 23 Çarpma işlemine sağ alttan başlarız. 3 tane 9 birliği çarparız. (3x9=27) Sonucun birler basamağını birler basamağındaki kısma yazar, onlar basamağındaki sayıyı onlar basamağının çarpımına elde olarak ekleriz. Devamında 3 tane 0 onluğu çarparız, Eldeyi ekleriz. 109 x 23 27 3x0=0 0+2=2 onluk Sonrasında 3 tane 1 yüzlüğü çarparız. Sonucu yüzler basamağındaki kısma yazarız. 3x1=3 109 x 23 327 Devamında 2 tane 9 birliği çarpar ve sonucun birler basamağındaki sayıyı onlar basamağının altına yazarız. (2x9=18) Eldeyi bir sonraki çarpıma ekleriz. Sonrasında 2 tane 0 onluğu çarpar eldeyi ekleriz. Sonrasında 2 tane 1 yüzlüğü çarparız. En son çarpım sonuçlarını toplarız. 109 x 23 327 +218 2507 10	We start the multiplication from the bottom right. We multiply three 9 units (3x9=27). We write it in the units place in the result. We add the number in the tens to the tens place. Then we multiply 0 tens by 3 and add the number in hand. 3x0=0, 0+2=2 tens. Then we multiply 1 hundreds by 3. We write the result in the hundreds place. 3x1=3. Then multiply 9 units by 2 and write it in the units place in the result. (2x9=18). We add what we have in hand to next operation. Then we multiply 1 hundreds by 2. Finally, we add the multiplication results.

Figure 4. The answer of a participant who made an incorrect/insufficient explanation in spite of solving the problem correctly using the algorithm.

As seen in Figure 4, the participant tried to explain the multiplication based on the positional notation, but expressed it as 20 9 units as 2 9 units. The reason for this finding may be the fact that 2 in the number 23 was not considered as 20.

Standard and non-standard models used by the participants in MT

Table 5 showed that the participants employed standard models all operation types.

Table 5. *Distribution of standard and non-standard models used by participants in MT*

Operation	Model	Examples	f	%

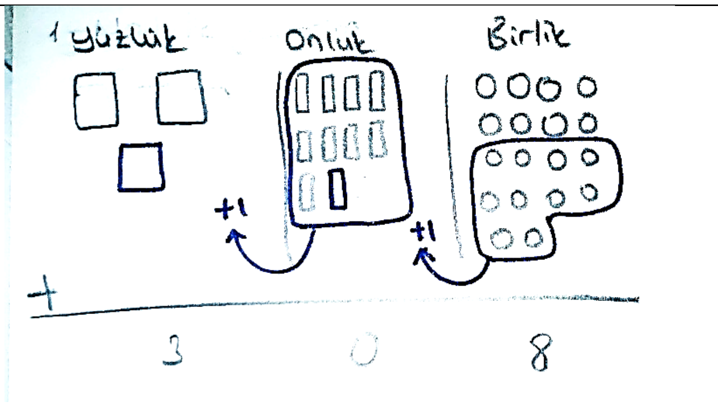
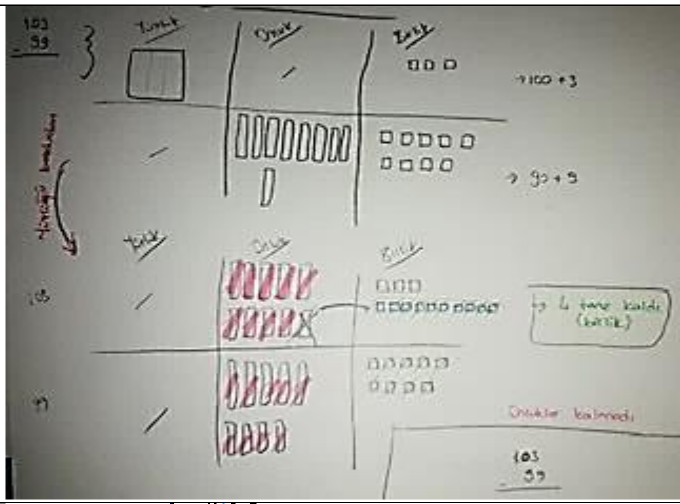
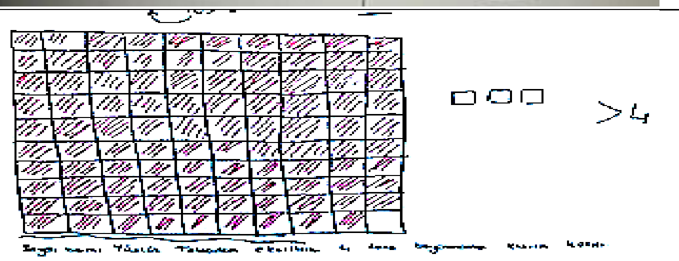
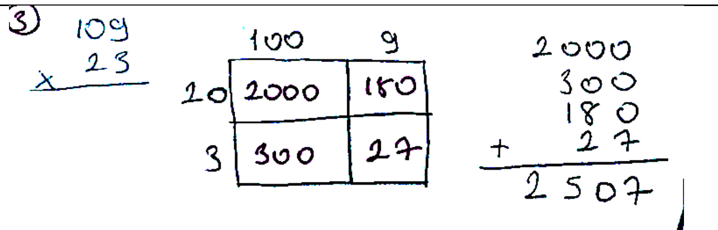
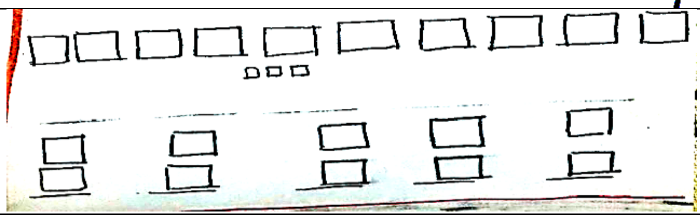
Addition	*Standard model* Unit, tens and hundreds blocks		101	100
Subtraction	*Standard model* Unit, tens and hundreds blocks		77	84
	Standard model hundreds table		15	16
Multiplication	*Standard model* Area model		86	100
Division	*Standard model* Unit, tens and hundreds blocks		67	100

Table 5 showed that the participants employed standard models all operation types. Non-standard models were not used in any operation type. Similarly, the most used standard model

in all operation types, except multiplication, was “unit, tens and hundreds blocks”. Unit, tens and hundreds blocks were the only models used by the participants in addition and division operations. In the subtraction, some of the participants also used the hundreds table.
The incorrect answers of the participants in modeling are presented below.

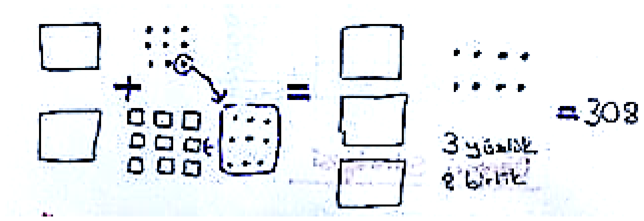


Figure 5. The answer of a participant who incorrectly modeled the question “109 + 99=?”
As seen in Figure 5, the participant was able to model the number 109 correctly by using blocks of 1 hundred and 9 units. However, the number 99 was incorrectly modeled by showing 1 hundred blocks and 9 ten blocks. The participant found the result of the addition as 3 hundreds and 8 units (308). Accordingly, it can be said that the participant’s modeling knowledge and content knowledge about positional notation were weak.

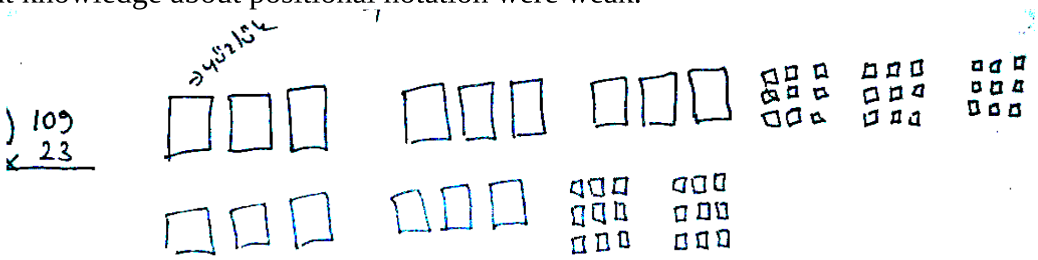


Figure 6. The answer of a participant who incorrectly modeled the question “109 x 23=?”
In Figure 6, if the number 109x23 is considered as the "number of objects and groups", either 23 of 109 or 109 of 23 should be modeled. However, the participant showed the number 109 not 23 times, but 15 times with blocks of hundreds and ones. Therefore, it can be argued that the participant’s modeling knowledge and content knowledge about multiplication were weak.

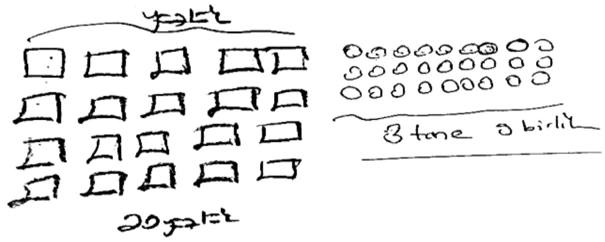
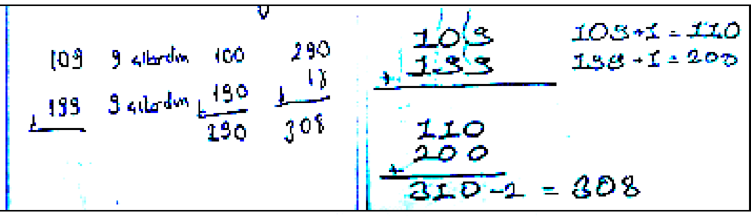
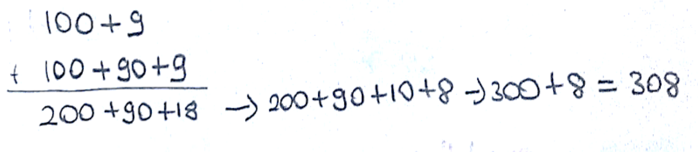
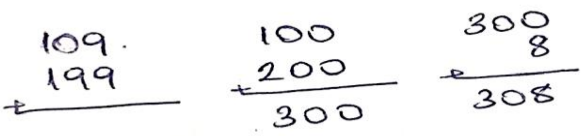
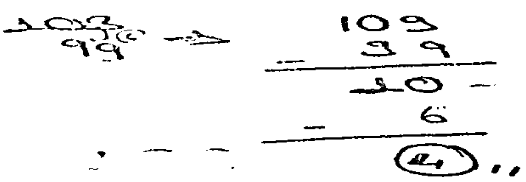
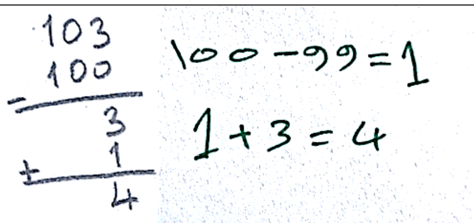
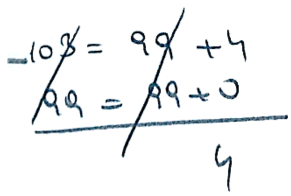
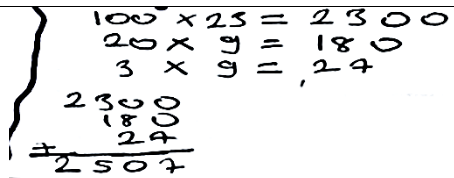
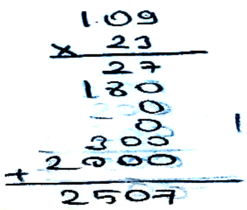
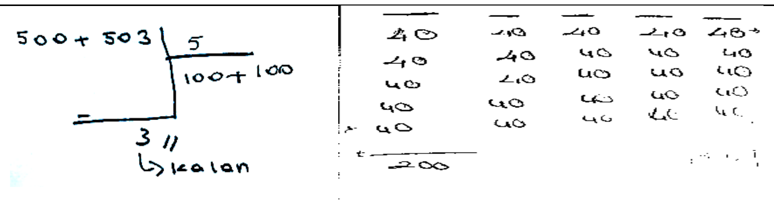
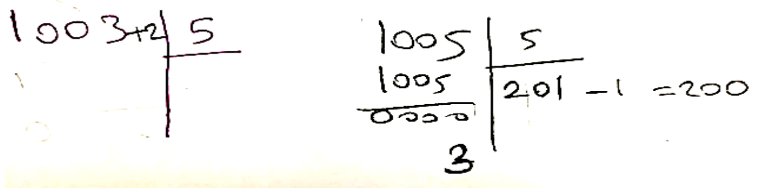
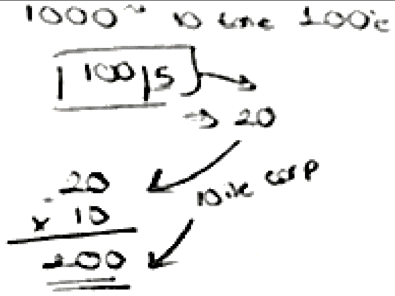


Figure 7. The answer of a participant who incorrectly modeled the question “109 x 23=?”
As seen in Figure 7, the participant did not consider 23x109 as 23 109 units, but instead modeled it as 20 hundreds and 3 9 units. The participant did not model 20 9 units and 3 hundred units and therefore did not include them in the solution. Accordingly, it can be said that the participant’s modeling knowledge and content knowledge about multiplication and positional notation were weak.
In sum, it can be said that participants commonly make incorrect modeling and draw the result by heart based on arithmetic operations.

Strategies used by the participants in AST

Table 6. *Distribution of the strategies used by participants in AST*

Operation	Strategy	Examples	f	%
Addition	Adding and subtracting the same number		28	29
	Adding hundreds, tens and units separately		37	38
	Rounding to hundreds		32	33
Subtraction	Adding and subtracting the same number		19	21
	Rounding to hundredths		62	69
	Splitting the number		9	10
Multiplication	Splitting the number		24	33
	Multiplying the digits separately and adding the results		49	67

Division	Splitting the number		35	81
	Adding numbers to the divisor and subtracting the numbers from the quotient		5	12
	Making use of multiples of numbers		3	7

It can be said that the participants preferred similar strategies in addition, subtraction, multiplication and division questions. Seven different strategies were used in all operations. Among the operation types, the most preferred strategies were " rounding to hundredths " and "splitting" strategies, whereas the least preferred strategy was "making use of multiples of numbers".

The participants mostly used "adding hundreds, tens and units separately" strategy (38%) in addition, and "rounding to hundredths" strategy (62%) in subtraction. Furthermore, the participants mostly used the "multiplying and adding the digits separately" strategy in multiplication (49%) and "splitting the number" strategy in division (35%).

The incorrect answers of the participants in using additional strategies are as follows:

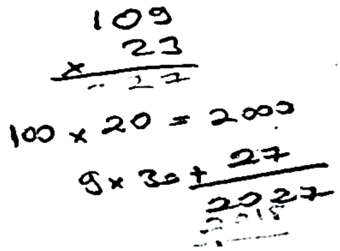


Figure 8. The answer of a participant who solved the question “109 x 23=?” incorrectly using additional strategies

As seen in Figure 8, the participant had misconceptions about the solution of multiplication. It was found that the participant generalized the rule of making operations between positional notations in addition and subtraction to multiplication. Thus, he/she multiplied only 3 by 9 and only 100 by 20. It can be said that the participants who provided such responses had poor additional strategy generation skills and content knowledge in multiplication.

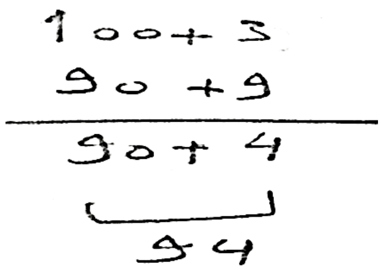


Figure 9. The answer of a participant who solved the question “103-99=?” incorrectly using additional strategies

As shown in Figure 9, in the operation 103-99, 103 was split into parts as "100+3" and 99 as "90+9". Then, the participant subtracted 9 from 3, but did not subtract tens from the tens digit. The second mistake the participants made was that he/she wrote 9 tens directly in the conclusion part, instead of subtracting 9 tens from 9 tens. As a result, it can be concluded that the participants who provided such responses had poor additional strategy generation skills and content knowledge in subtraction.

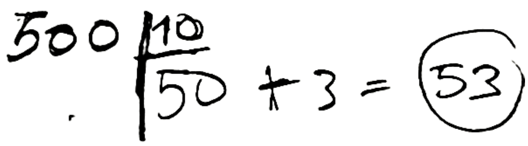


Figure 10. The answer of a participant who solved the question “1003 ÷ 5 =?” incorrectly using additional strategies

As seen in Figure 10, the participant split the number 1003 into parts as “500+500+3”. However, instead of dividing the numbers by 5, the participant divided the numbers by 10. It is seen that the participant did not know making division using additional strategies and the interpretation of the remainder in division. Therefore, it can be argued that the participants who provided such responses had poor additional strategy generation skills and content knowledge in division.

4.Conclusion

The aim of this study was to examine the pre-service classroom teachers’ knowledge on four operations. The data were collected through three data collection tools: AT for operational knowledge, MT for modeling knowledge, and AST for additional strategy knowledge. Participants were generally more successful in addition and had more difficulty in division in all tests. Accordingly, the success order of operation types in all tests was as follows: addition, subtraction, multiplication and division. In all tests, the participants’ knowledge was good in addition and poor in multiplication and division.

The participants had over 90% of success in AT. In this context, it can be said that the participants’ four operations knowledge was at a good level. The participants who provided incorrect answers in AT had procedural errors and difficulty in providing instructional explanations based on the positional notation. According to Baki (2013), the reason for this finding is that pre-service teachers use rote rules and have difficulty in structuring the operations according to the positional notation. In this sense, it can be put forward that the participants had difficulty in applying the knowledge they learned in the Mathematics Teaching-I course and their content knowledge was weak in terms of instructional explanations based on positional notation.

The success rate in MT was between 61% and 92%. Rather than giving wrong answers, the participants did not provide any answers in modeling questions. This finding is in line with those in previous studies (Aksu & Konyalıoğlu, 2015; Bayezit et al., 2011; Duran et al., 2016; Kertil, 2008; Korkmaz, 2010; Dede and Yılmaz, 2013). The participants had the most difficulty in modelling division. Similarly, Tekin

Sitrava et al. (2020) stated that pre-service teachers' modeling knowledge on division was not sufficient, pre-service teachers did not know how to use conceptual knowledge and they had procedural knowledge. The reason for this finding may be that pre-service teachers had limited conceptual and operational understanding of division, as observed in AT. In addition, the models used in division were fewer compared to addition, subtraction and multiplication, which may be due to difficulty in modeling the division. The models used by the participants showed that they preferred standard models in all operation types and never use non-standard models. In this context, it can be argued that pre-service teachers had poor knowledge of non-standard modeling in four operations. The reason why the participants failed to use non-standard models may be that they could not generate their own modeling or that the use of non-standard models was not favored in teaching. The participants mostly preferred to use "one, ten and hundred" blocks in addition, subtraction and division operations, and area model in multiplication. Few participants used the hundreds table in subtraction. The participants having difficulties in modeling provided incomplete modeling or could not model the operations due to their poor content knowledge. The participants generally thought operationally and generate models accordingly. Another reason why the participants had difficulties in modeling may be that four operations are usually taught only with algorithms in schools. Pre-service teachers who learn the four operations only operationally and cannot understand them conceptually will have difficulty in associating these operations with a model. In this regard, NCTM (2000) stated that students who can model operations will understand concepts better. In order for primary school students who are in the concrete operational stage to understand mathematical concepts, they need to be associated with concrete objects and models (Tuna & Serin, 2019). Therefore, studies should be carried out to increase the modeling knowledge of pre-service teachers.

The success rate in AST was between 43% and 88%. This finding is in line with those in the literature (Bahar, 2019; Bozkurt & Yavaşça, 2021; Gürbüz & Güder, 2016; Şengül & Gülbağcı Dede, 2014; Yeşildere & Akkoç, 2010). As in other tests, division was the most problematic type of operation that in EST. The success rate was 43% in division. Accordingly, it can be said that the participants' additional strategies generation knowledge was weak in division. This finding may be due to participants' weak conceptual and operational knowledge about division, as in AT and MT. The participants used seven different strategies in solving the four operation questions. Participants preferred "rounding the number to one hundred" and "dividing the number" strategies the most, and "making use of multiples of the number" the least. The fact that four operations is taught using only algorithm and the use of additional strategies is not favored in teaching may be the reason why the participants used similar strategies.

5.Suggestions

As a result, it can be said that the participants solved the four-operation questions operationally and had difficulty in solving them with a model or an additional strategy. Pre-service teachers having poor modeling and additional strategies knowledge cannot teach these topics to their future students. In mathematics teaching, modeling and additional strategies are used in all the achievements from the 1st grade to the 4th grade in primary school. The four operations are the foundation of mathematics. The pre-service teachers who cannot use modeling and additional strategies in four operations are not expected to use this knowledge in future mathematics topics. In this context, teacher training institutions should restructure their curricula and provide pre-service teachers with opportunities to practice. Class hours for teaching mathematics should be increased, and different measurement and evaluation techniques, which allow pre-service teachers to explain their operations, should be included in the courses.

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