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INVESTIGATING THE IMPACT OF INTERVENTIONS USING CONCRETE AND VIRTUAL MANIPULATIVES ON 3 TH GRADE STUDENTS' FRACTION CONCEPT AND MOTIVATION

(*Research article*)

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Abstract

The purpose of this study is to examine the effect of using concrete and virtual manipulatives relevant fractions subject to 3rd grade students' understanding of fractions and motivation towards mathematics lessons. The study group consists of 61 students studying in three different classes at the 3rd grade level of a private primary school in Turkey. A quantitative research method, pretest and posttest, and a quasi-experimental research design including two experimental and one control group were adopted. The study was carried out by the experimental-1 group with concrete manipulative-assisted training, the experiment-2 group with virtual manipulative-assisted training, and the control group with the traditional teaching in the mathematics curriculum. Data collection tools were the Fraction Comprehension Test (FCT) developed originally with a cronbach alpha reliability coefficient of 0.874 and the Mathematics Lesson Motivation Scale (MLMS) for primary schools was used. One-way MANOVA was used for analysing the data. As far as the research findings; using manipulatives makes a statistically significant difference in the understanding of fractions of 3rd grade students ($F_{(2\text{-}60)} = 9.171$, $p<.05$), but it was detected that there was not statistically important difference ($F_{(2\text{-}60)}= 0.163$, $p>.05$) in motivation for mathematics lessons. It is seen that 24% of the change in comprehension is caused by using manipulative. Concrete manipulative use and virtual manipulative use have no superiority in terms of both fraction comprehension and mathematics motivation when compared to each other. It can be suggested that using manipulatives as complementary to each other may give better results.

Keywords: Concrete manipulative, fraction comprehension, mathematics education, motivation, virtual manipulative.

1. Introduction

Students acquire mathematical knowledge by concretizing and making sense with effective mathematics education in primary school. Because a student at the age of primary school needs experience with materials to learn mathematical concepts. In school mathematics, students begin to learn mathematics using gamified real-life situations, concrete objects and visuals, and then they move on to using mathematical symbols. Because of this, it is recommended that students use manipulative objects to learn new mathematical concepts (Clements & McMillen, 1996; Hiebert & Carpenter, 1992; Hsiao, 2001; McClung, 1998, p.2; McNeil & Jarvin, 2007). Therefore, manipulatives have a significant role in mathematics teaching in terms of enabling students to reach different related representations of mathematical expressions.

The concept of fraction in basic education is difficult for pupils to comprehend (Dorgan, 1994; Hansen, 2014). The reasons for these difficulties can be given as an example that pupils encounter the notion of fraction less in their daily life than natural numbers (Suh, 2005), and a fractional expression shown as a/b contains different meanings (Kieren, 1993; Olive, 1999). In order to better grasp the concept of fraction and minimize the difficulties experienced, it is suggested by researchers to use manipulative objects in fraction teaching (Brown, 2007; Hansen, Mavrikis & Geraniou, 2016; Önver, 2019; Reimer & Moyer, 2005; Suh, 2005). In addition, it was stated that teachers mostly need manipulatives in teaching the concept of fraction (Çetin, et al.,2019).

In literature, we rarely encounter that interventional (experimental-action research, etc.) studies that compare concrete and virtual manipulatives are mostly used at the secondary school level (Brown, 2007; Lee & Chen, 2015; Olkun, 2003; Önver, 2019; Şahin, 2013; Takahashi, 2002; Yolcu, 2008) at the high school level and above (Gülkılık, 2013; Öz, 2012) and again rarely at the primary school level (Ross, 2008; Suh, 2005). Most of these studies have focused on the cognitive domain and researched academic achievement. When the research results are evaluated in general, there is also no clear conclusion about which type of manipulative (concrete - virtual) is more effective. At the basic education level, there are very few experimental studies investigating the effects of concrete and virtual manipulatives in terms of both cognitive (comprehension) and affective (motivation) issues in teaching the concept of fraction. For this reason, we have purposed to compare motivation towards mathematics and fraction comprehension and lessons of the groups that use concrete manipulatives / virtual manipulatives and those who do not use manipulative at 3rd grade level. Thus, we estimate contributing to the literature on manipulative use in mathematics education and shedding light on researchers / practitioners working on this subject. This study aims to investigate both comprehension (cognitive) and its effects on motivation (affective) using concrete manipulative / using virtual manipulative / not using manipulative at the elementary school 3rd grade level, which is seen as the beginning level for the development of basic fraction concepts (NCTM, 2006), and the decrease in motivation towards mathematics lesson begins (Harter, 1981). Also, this study has subjected to the primary school, where the foundations of students' education life were laid, the effectiveness of concrete and virtual manipulatives used in teaching fraction concepts, which are considered important in mathematics education programs, and constitute the basis of many mathematics topics, and which students have misconceptions.

1.1. Concrete and Virtual Manipulatives

Manipulative is defined as "objects designed to represent mathematical ideas in a clear and concrete way" (Moyer, 2001). In fact, the theoretical foundations of manipulative date back to the past (Bruner, 1966; Dienes, 1973; Piaget; 1965). These dynamic objects that provide a good opportunity for the creation of mathematical knowledge can be both concrete (tactile, real, sensory, real) manipulatives (Clements, 1999) and virtual (interactive, web-based visual) manipulatives that support the transition between visual representations and soft information (Moyer, Bolyard & Spikell, 2002; Volk, Cotič, Zajc & Starčič, 2017). In some studies, that compared the effects of concrete and virtual manipulatives on academic achievement, a significant difference was found between both types of manipulatives (Gülkılık, 2013; Lee & Chen, 2015; Ross, 2008; Suh, Moyer & Heo, 2005; Şahin, 2013; Takahashi, 2002; Yolcu, 2008). On the other hand, it was found insignificant difference between the two types of manipulative using (concrete-virtual) in terms of academic achievement (Moyer-Packenham, et al., 2013; Suh, 2005) or it is detected a very low level of difference in favor of the virtual manipulative group (Deliyianni, Michael & Pitta- Pantazi, 2006; Moyer, Niezgoda & Stanley, 2005). Trespalacios (2008) disagrees with this opinion. In Lee & Chen's (2015) study with 5th grade students, it was stated that virtual manipulatives are as effective in understanding fractions as concrete manipulatives, and virtual manipulatives make learning more enjoyable compared to concrete manipulatives. In a similar study, it was emphasized that both types of manipulative in teaching geometry contribute to learning in different directions and it was emphasized that both manipulatives should be used in classrooms in a way that complements each other in order to maximize students' learning (Takahashi, 2002). The reason for achieving higher results in using virtual manipulatives when comparing the use of concrete manipulatives in fraction teaching; it was emphasized that the step-by-step process is due to the ease of implementation and instant feedback (Suh, 2005). However, according to the results obtained from Brown's (2007) study concluded that concrete manipulatives are more effective in student performance than virtual manipulatives. On the other hand, a recent study revealed that manipulative use does not have a statistically significant difference in mathematics achievement and motivation (Önver, 2019).

In the study which only virtual manipulatives were included in 3rd grade fraction teaching was observed that there was a statistically significant increase in the posttest scores of the conceptual knowledge test of students using virtual manipulatives (Reimer & Moyer, 2005). The result obtained from the attitude questionnaires and student interviews; it was stated that virtual manipulatives provide instant feedback to students and help students learn more about fractions, and they are used easier and faster than using paper and pencil. In another study investigating the effect of using only virtual manipulative in teaching the subject of fraction at the 3rd grade Trespalacios (2008) revealed that learning activity using virtual manipulative has no superiority. On the other hand, Speer (2009) stated that the use of virtual manipulative can be seen as complex; negative aspects such as misuse of virtual manipulative are emphasized. Unlike the methods of these studies, studies synthesizing previous studies in this field (Carbonneau, Marley & Selig, 2013; Domino, 2010) stated that using manipulative in mathematics teaching results in better mathematics success than traditional teaching methods. In summary, the research is about the fact that concrete-virtual manipulative supported mathematics applications provide the opportunity to create conceptual knowledge. With this, the studies did not deal with many mathematics subjects in terms of multiple skill areas (cognitive-affective-psychomotor) at the primary school level. Each study usually only deals with one grade level and topic. This research is limited to the 3rd grade level and fraction subject. The strength of the research is that it is examined the effectiveness of an intervention, both cognitively and affectively, by arranging two experiment groups and a control group. As a result of these interventions, the comparison of the two experiment groups and the control groups is to reveal a scientific result about which method (concrete manipulative-virtual manipulative intervention) is effective.

1.2. Research Question

Do the use and non-use of concrete and virtual manipulatives in 3rd grade fractions affect students' level of fraction comprehension and their motivation towards mathematics lessons?

1. Is there a statistically significant difference between the mean scores of the Fraction Comprehension Test and the Mathematics Lesson Motivation Scale of the experiment-1 using concrete manipulative, experiment-2 using virtual manipulative, and the control group with no manipulative use?

2. Is there a significant difference between the Fraction Comprehension Test pretest-posttest and the Mathematics Lesson Motivation Scale pretest-posttest scores of the experiment-1 using concrete manipulative, experiment-2 using virtual manipulative, and the control group with no manipulative use?

2. Method

In this research, quasi-experimental design, which is one of the quantitative research methods, was adopted to examine the effect of concrete and virtual manipulative supported teaching practices on 3rd grade students' fraction comprehension and motivation. This method is a research method used to examine any event, phenomenon, person, object and factor, find cause-effect relationships between parameters and measure the results by comparing them with each other (Cohen, Manion & Morrison, 2000). In this direction, there are three study groups, two of which are experiment groups and one is a control group, within the scope of the research. The first experiment group was determined as the group in which concrete manipulative supported fraction teaching was performed, the second experiment group was given virtual manipulative supported teaching, and the control group was determined as the group without any intervention. Within the content of experimental design, its impact on students' motivation towards fractional comprehension and mathematics lesson will be considered as a dependent variable, concrete and virtual manipulative use case as independent variables.

The experimental design of the research is shown in Table 1.

Table 1. *Research design*

Group	PreTest	Practice	Final Test
G1 (n=21)	Fraction Comprehension Test and Mathematics Lesson	X1	Fraction Comprehension Test and Mathematics Lesson Motivation
G2 (n=20)		X2	
G3 (n=20)		X3	

G1: Experiment Group 1; G2: Experiment Group 2; G3: Control Group

X1: Concrete Manipulative Practice; X2: Virtual Manipulative Practice; X3: Traditional teaching

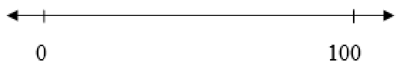
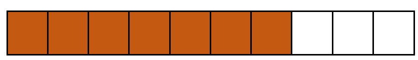
2.1. Participants

The working group of the research consists of 3rd grade students studying in a private primary school in Turkey in the Central Anatolia Region. In the study group, there are 61 students in total, 21 students who make up the first experiment group (using concrete manipulative), 20 students who make up the second experiment group (using virtual manipulative), and 20 students who make up the control group (traditional education). The study group was randomly selected using easily accessible sampling, one of the targeted sampling methods.

2.2. Data Collection

In the research, the comprehension test was used developed originally by a researcher who specialized in 3rd grade fraction subject. For this, "Fraction Comprehension Test (FCT) was designed to measure, the achievements within the subject of fractions in the mathematics program (MoNE, 2018) and the meanings of fractions (part-whole, division, ratio, measurement and processor) and fraction models (area, length, set) in the literature of mathematics education (van de Walle, 2004). In the process of developing fraction comprehension test, 20 items were prepared by taking the opinions of a classroom teacher, a mathematics teacher and a specialist in mathematics education. In order to determine the validity and reliability level of the prepared test, a pilot study was conducted with 76 students before the application. As a result of the pilot study, item analyzes were made, fundamental corrections were made to the items, and the comprehension test was finalized. FCT was carried out on the control group and experiment groups before and after the practice. While examining the results of the fraction comprehension test, it was evaluated that 3 points if the transaction and the result are correct, 2 points if the result is correct but the result is incorrect, '1' point if the transaction and result are partially correct and the result is incorrect, and '0' point for the wrong or blank answers. The total score each student got was calculated. Cronbach alpha reliability coefficient of FCT was calculated as 0.874. For the content validity of FCT, the question and outcome association in the test is presented in Table 2.

Table 2. *The correlation between the questions in the comprehension test and the learning outcomes in the mathematics curriculum*

Achievement	Question Number	Sample Question
M.3.1.6.1. Uses fraction representations of whole, half and quarter models.	1,2,3	How many whole apples would a child eat a half and six-quarter apple?
M.3.1.6.2. Separates a whole into equal parts and indicates that each of the identical parts is a unit fraction.	4,5,6	Which one is bigger bread of 1/2 or 1/3 ?
M.3.1.6.3. Explains the relationship between numerator and denominator.	7,8,9	In a school, the number of A class students is 4/6, and B class students 1/3. Which class has more students?
M.3.1.6.4. Shows the unit fractions of fractions with a dominator of 10 and 100.	10,11,12	What is the unit fraction of 28/100? Show it on the model. 
M.3.1.6.5. Determines the specified unit fraction of a plurality.	13,14,15,16	Berfu, Ceren and Yiğit have 100 Turkish Liras. 3 friends made shopping at the market, Berfu paid 1/2, Yiğit 1/4 and Ceren 1/5 at the cash register. Accordingly, who has spent the most and who has spent the least?
M.3.1.6.6. Obtains smaller fractions from the denominator of the numerator.	17,18,19,20	Show the model on the number line. 

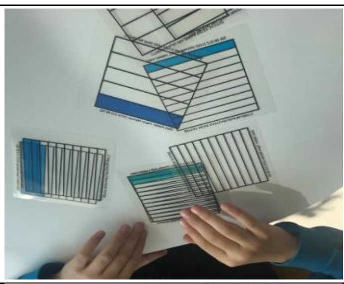
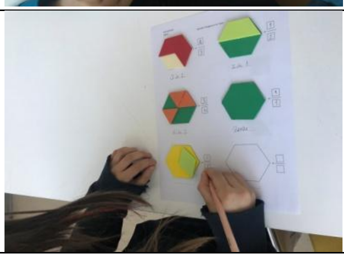
In addition, the “Mathematics Lesson Motivation Scale (MLMS) for Primary School 3rd and 4th Grade Students” developed by Balantekin & Oksal (2014) was used. The scale is a 5-point Likert type and consists of Strongly Agree (5), Agree (4), Somewhat Agree (3), Disagree (2), Strongly Disagree (1). MLMS, the Cronbach alpha internal consistency coefficient calculated by Balantekin and Oksal (2014) for the Extrinsic Motivation factor ($\alpha = .78$); It was calculated as ($\alpha = .71$) for the no motivation factor and ($\alpha = .61$) for the Intrinsic Motivation factor. Within the scope of the research, the Cronbach alpha was not calculated and the reliability coefficient of the mentioned research was accepted. MLMS has three factors (extrinsic motivation, lack of motivation and intrinsic motivation) and the items belonging to the no motivation factor (6,7,8,9 and 10) were coded as inverse items in this study. While analyzing these coded data, the total score of MLMS was taken as a basis. The scale includes items such as "Mathematics lesson interests me", "I get bored quickly in math lesson", "I get happy while studying math lesson".

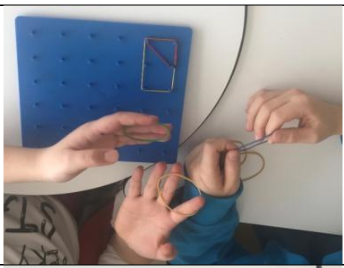
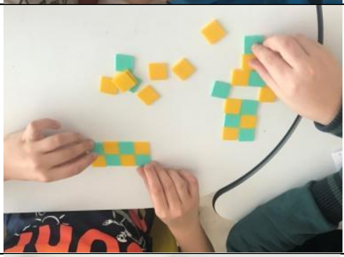
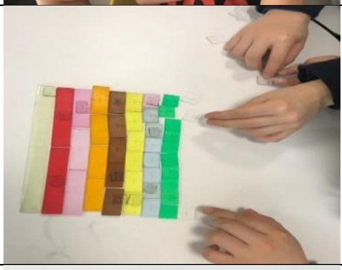
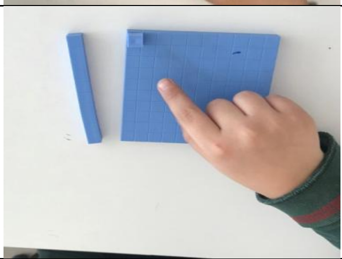
2.2.1. The implementation of the intervention

The classes to be applied were determined after obtaining the necessary permissions from the relevant institutions with the approval of the ethics committee by applying to the Scientific Research Ethics Committee. Accordingly, one of the 3rd graders was chosen as the experiment group for concrete manipulative-supported teaching, one as the experiment group for which virtual manipulative-supported teaching and the other as the control group without intervention. On the subject of fractions, plans suitable for teaching with manipulative support were prepared for the experiment groups and the application was made for eighteen hours. At the beginning of the process, "Fraction Comprehension Test" and "Mathematics Lesson Motivation Scale" pretests were applied to all students in the groups.

The researcher applied a concrete manipulative-supported teaching plan to the first experiment group, and a virtual manipulative-supported teaching plan to the second experiment group. Any intervention was not made to the control group. The practitioner researcher carried out the applications in the mathematics lessons of two different branches on different days and hours. Finally, all students in the group are again carried out the final test of “Fraction Comprehension Test and Mathematics Lesson Motivation Scale. Table 3 shows the sample manipulatives used in the first experiment group that used concrete manipulatives.

Table 3. *Concrete Manipulatives*

Manipulative	Picture	Explanation
Fraction cards		It consists of blocks divided into equal parts of 1 (full), 2, 3, 4, 5, 6, 8, 10 and 12, and it is a total of 69 cards. Each of the cards is painted in a different number of pieces, and the cards that are divided into the same number of identical pieces are of the same color.
Pattern blocks		Pattern blocks are plastic materials consisting of an equilateral triangle, right triangle, square, rectangle, hexagon and trapezoid shaped parts.

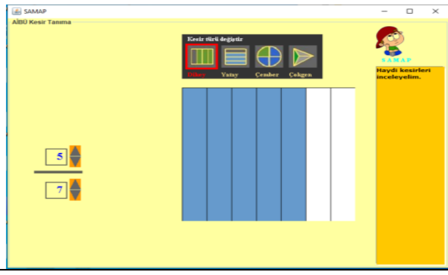
Geoboard		It is square shaped, material consisting of 6 pins on a plate made of plastic, vertically and horizontally at 3 cm intervals. Packed tires are used to form shapes on them.
Counters		It is made of small, colored plastic.
Cuisenaire rods		It is a material made of wood that has fragments of ten units' lengths from one unit. Each unit of length is shown in a different color.
Fraction Sets		Fraction Sets are materials consisting of plastic strips that show a whole divided into 2, 3, 4, 5, 6, 8 and 10 respectively.
Ten Base Block		It is a material made of plastic material showing the numbers 1, 10, 100 and 1000.
Circular fraction sets		They are materials made of wood formed by dividing a whole circumferential region considered as a whole into 2, 3, 4, 5, 6, 9, 10 and 12 equal parts, respectively.

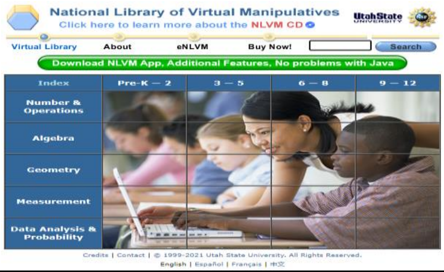
Concrete manipulatives shown in Table 3 were provided and they were used by students in classroom practices.

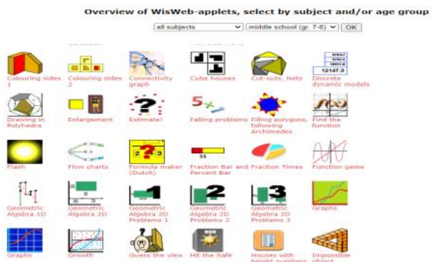
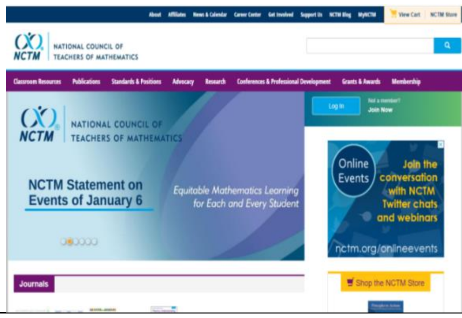
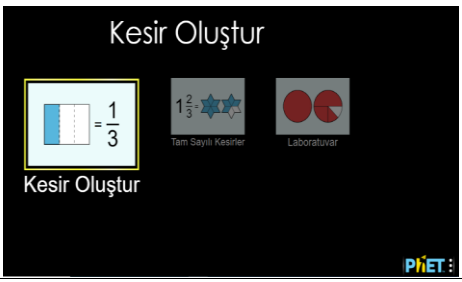
In Table 4, sample manipulatives used in the second experiment group using virtual manipulative are given.

Table 4. *Virtual manipulatives*

<table>
<tr><th>Manipulative-Explanation</th><th>Picture</th></tr>
<tr><td>Virtual Mathematics Manipulative Set (VMMS): It is a virtual manipulative developed for mathematics lessons.

http://erolkarakirik.com/samap/</td><td></td></tr>
<tr><td>Geogebra Virtual Manipulatives: It is a dynamic math software with multi-platform support and open-source code. https://www.geogebra.org/?lang=tr</td><td></td></tr>
<tr><td>National Library of Virtual Manipulatives (NLVM): It is a web-based, interactive virtual manipulative for mathematics lessons.

http://nlvm.usu.edu/</td><td>
</td></tr>
<tr><td>Wisweb is a virtual manipulative for math lessons.

http://www.fi.uu.nl/wisweb/en/home/welcome.html</td><td>
</td></tr>
<tr><td>National Council of Teachers of Mathematics-Illuminations (NCTM Illuminations): It was developed by the American Teachers Association (ATA). https://www.nctm.org/</td><td>
</td></tr>
<tr><td>PhET Colorado: Simulations of chemistry, biology, physics, earth sciences and mathematics provide representations of interactive research-based physical phenomena. https://phet.colorado.edu/tr/</td><td>
</td></tr>
</table>

Students were given preliminary information about the virtual manipulatives shown in Table 4 and they were used in the computer classroom.

2.3. Data Analysis

The data of the research were collected using the Fraction Comprehension Test of 20 questions and the Mathematics Lesson Motivation Scale form of 14 items. SPSS 25.0 was used for the analysis of the obtained data. Since the number of participants in the study groups that the application was carried out was less than 30, Kolmogorov-Smirnov test analyzes were examined. It was observed that the experiment-1 group ($p > .05$), experiment-2 group ($p > .05$) and the control group ($p > .05$) were normally distributed, respectively. Also the skewness and kurtosis values are suitable for the normality (-1.5 between 1.5). Because the normality assumptions of the data were realized, the following parametric tests were preferred.

In this study, one independent variable (manipulative using) and more than one dependent variable (its effect on 3rd grade students' motivation of mathematics lesson and fractional comprehension) will be investigated. Therefore, one-way MANOVA was used to determine whether there was a statistically significant difference between the pretest and posttest scores of the experimental and control groups. On the other hand, dependent samples t-test was used to examine the difference between the experiment-1, experiment-2 and control group students' 3rd grade fraction comprehension pretest-posttest scores and mathematics lesson motivation scale pretest-posttest scores.

3. Findings

The research states that "is there a statistically significant difference between the mean scores of the post test scores of the experiment-1 (concrete manipulative group), experiment-2 (virtual manipulative group) and control groups in which different applications were made from the Fraction Comprehension Test and Mathematics Lesson Motivation Scale? findings regarding the problem are given below. Descriptive statistics regarding the post test scores of the participants are shown in Table 5.

Table 5. *Descriptive statistics on the final test scores of the participants*

Variable	Group	N	$\bar{X}$	sd
Comprehension Final Test	Experiment 1	21	41.809	13.463
	Experiment 2	20	38.800	14.898
	Control	20	26.250	6.850
Motivation Final Test	Experiment 1	21	39.190	6.431
	Experiment 2	20	37.700	8.736
	Control	20	39.400	9.789

When Table 5 is examined, looking at the average values of the groups' "Fraction Comprehension Test (FCT)" posttest score, it appears to be higher Experiment 1 using concrete manipulative ($\bar{X}$= 41.809; sd = 13.463) and the mean scores ($\bar{X}$ = 38.800; sd= 14.898) of the experimental-2 groups in which virtual manipulative were used according to the mean score ($\bar{X}$= 26.250; sd= 6.850) of the control group without any intervention. When looking at the average values of the "Mathematics Lesson Motivation Scale (MLMS) " posttest score, the average score of the control group, it is seen that ($\bar{X}$= 39.400; sd= 9.789) experiment-1 ($\bar{X}$= 39.190; sd= 6.431) and experiment-2 ($\bar{X}$= 37.700; sd= 8.736) groups are higher than the average score.

One-factor MANOVA was used to test whether the difference between the mean scores of experiment-1, experiment-2 and control groups was understandable. Whether the variance-covariance homogeneity, which is one of the prerequisites of the MANOVA analysis, was achieved, was decided by looking at Box's M statistics (p> .05). In addition, it was decided according to the Levene test (p> .05) that the variance equivalence condition was fulfilled. MANOVA results regarding whether the students in the Experiment-1, Experiment-2 and Control groups show a significant difference in terms of the mean FCT posttest score and MLMS posttest score are shown in Table 6.

Table 6. *One-factor MANOVA results on the manipulative use of posttest scores of 3rd grade students' motivation for fraction comprehension and mathematics lesson*

Independent Variable	Dependent Variable	Sum of Squares	df	R Squares of mean	F	p	η2
Use of Manipulative	Comprehension	2762.074	2	1381.037	9.171	.000	.240
	Motivation	16.952	2	8.476	.163	.850	.006

When Table 6 is examined, one-factor analysis of variance results are seen to determine the effect of manipulative use on 3rd grade students' motivation for understanding fractions and mathematics lesson. FCT Final Test possess (Wilks Lambda (λ) = 0.751; $F_{(2-60)}$ = 9.171; p <.05; η2 = .240), MLMS Final Test possess (Wilks Lambda (λ) = 0.879; $F_{(2-60)}$ = .163; p> .05; η2 = .006). According to the MANOVA analysis findings; the use of manipulative was found to be a factor ($F_{(2-60)}$ = 9.171; p <.05) that significantly differentiates 3rd grade students' understanding of the subject of fractions. According to this; 24% of the change of 3rd grade students in understanding the subject of fractions is due to using manipulative. However, it can be said that manipulative use is not a factor ($F_{(2-60)}$ = .163; p> .05) that significantly differentiates 3rd grade students' motivation towards mathematics lesson.

Post-hoc test results, one of the multiple comparison tests, were examined in order to determine which groups the difference was between for the values found significant in the MANOVA table. Turkey's test result is given in Table 7 below. Accordingly, it can be determined among which groups the difference in FCT scores.

Table 7. *Difference between groups*

Source of Variance	Sum of Squares	df	Means of Squares	F	p	Significant Difference
Comprehension Final Test	2762.074	2	1381.037	9.171	.000 .006	D1-K, D2-K

When Table 7 was examined, it was found that the differences between the groups were caused by the differences between the experimental-1 and control groups (p = .000) and also between the experimental-2 and control groups (p = .006). According to these results, it was found (p <.05) that the mean FCT Final Test score of the experimental-1 group ($\bar{X}$ = 41.809; sd= 13.463) was significantly higher than the mean post test score ($\bar{X}$ = 26.250; sd= 6.850) of the control group.

Similarly, it was found (p <.05) that the mean post test score of the experimental-2 group ($\bar{X}$ = 38.800; sd= 14.898) was significantly higher than the mean FCT final test score of the control group ($\bar{X}$ = 26.250; sd= 6.850). However, it was observed that there was no statistically significant difference between experimental-1 group and experiment-2 groups' mean FCT posttest scores (p> .05). As can be understood from here; shows that the use of any manipulative (concrete or imaginary) has an effect on students' comprehension of the fraction. However, the use of concrete and virtual manipulative does not have an advantage over the other in fractional comprehension. The second sub-problem of the study was "Is there a significant difference between Fraction Comprehension Test pretest-posttest and Mathematics Lesson Motivation Scale pretest-posttest scores of Experiment-1 (concrete manipulative), experiment-2 (virtual manipulative) and control groups?" about findings regarding the problem are given below.

The descriptive and t test results comparing the Fraction Comprehension Test and the Mathematics Lesson Motivation Scale scores of the students before and after the study are given in Table 8.

Table 8. *Dependent sample t-test results regarding fraction comprehension scores of the groups.*

	N	$\bar{X}$	ss	sd	t	p
Experiment1						
Pretest	21	15.714	10.364	20	-10.289	.000
Posttest	21	41.809	13.463			
Experiment2						
Pretest	20	12.850	13.144	19	-12.651	.000
Posttest	20	38.800	14.898			
Control						
Pretest	20	18.150	6.830	19	-2.447	.000
Posttest	20	26.250	6.850			

When Table 8 is examined, it has been found ($t_{(20)} = -10.289$; $p <.05$) that the difference between the mean scores of FCT pretest and posttest after concrete manipulative supported mathematics education of the students in the experimental-1 group is statistically significant, Accordingly, it can be said that concrete manipulative supported fraction teaching has an effect on the experimental-1 group students' understanding of fraction.

After the virtual manipulative assisted mathematics education of the students in the Experiment-2 group, the difference between the mean scores of the FCT pretest and posttest was found to be statistically significant ($t_{(19)} = -12.651$; $p <.05$). Accordingly, it can be said that virtual manipulative assisted fraction teaching affected the experimental-2 group students' fraction comprehension.

After the mathematics education of the students in the control group, which was carried out in accordance with the activities in the current mathematics teaching program, the difference between the mean scores of the FCT pretest and posttest was found to be statistically significant ($t_{(19)} = -2.447$; $p <.05$). Accordingly, it can be said that the fraction comprehension scores of the students increased significantly after the traditional fraction teaching as in the other groups.

The results of the t-test made for the significance of the difference between the MLMS pretest-posttest average scores of the groups are given in Table 9.

Table 9. *Dependent sample t-test results for motivation scores for mathematics lessons belonging to the groups*

		N	$\bar{X}$	ss	sd	t	p
Experiment1	Pretest	21	38.095	6.541	20	-.588	.563
	Posttest	21	39.190	6.431			
Experiment2	Pretest	20	37.000	8.926	19	-.499	.624
	Posttest	20	37.700	8.736			
Control	Pretest	20	38.150	5.869	19	-.747	.464
	Posttest	20	39.400	9.789			

When Table 9 is examined, it was found that the difference between the mean scores of MLMS pretest-posttest after concrete manipulative supported mathematics education of the students in the experimental-1 group is not statistically significant. ($t_{(20)} = -.588$, $p> .05$) Accordingly, it cannot be said that the use of concrete manipulative affects students' motivation towards mathematics lesson.

After the virtual manipulative assisted mathematics education of the students in the Experiment-2 group, it was found that the difference between the MDMÖ pretest-posttest mean scores was not statistically significant. ($t_{(19)} = -.499$; $p> .05$). Accordingly, it cannot be said that the use of virtual manipulative affects students' motivation towards mathematics lesson.

It was found that the difference between the mean scores of MSML pretest and posttest after mathematics education conducted in accordance with the activities in the current mathematics teaching program of the students in the control group was not statistically significant. ($t_{(19)} = -.747$; $p> .05$). Accordingly, no change was observed in the motivation of the control group students towards mathematics lesson. In other words, it can be said that any manipulative use has a significant effect on students' comprehension of fraction, but does not have a positive effect on their motivation towards mathematics lesson.

4. Discussion and Conclusion

In this study, it is aimed to compare the fraction comprehension and motivation for the mathematics lesson of the groups who use concrete manipulatives at the 3rd grade level, and those who use virtual manipulatives and those who do not, using an experimental method. It was determined that there were statistically significant differences in favor of the experimental groups between the group in which concrete manipulatives were used, and the group in which virtual manipulatives were used and the control group students in the comprehension of fractions by 3rd grade students. It is seen that this finding is similar to the results of the studies conducted on the subject of fractions with experimental design (Brown, 2007; Lee & Chen, 2015; Reimer & Moyer, 2005; Suh, Moyer & Heo, 2005). When studies in different subjects of mathematics (Gülkılık, 2013; Ross, 2008; Şahin, 2013; Takahashi, 2002; Yolcu, 2008) are examined; the fact that manipulative use has a positive effect on learning is similar to the results of the research. In addition to these studies, the positive findings of meta-analysis studies (Carbonneau, Marley & Selig, 2013; Domino, 2010) on the effect of manipulative use in mathematics teaching are consistent with the results of the research. When the experimental groups are compared with each other; it is seen that there is no statistically significant difference between fractional comprehension of the group in which concrete manipulatives are used and fraction comprehension of group in which virtual manipulatives are used. This result seems to be in line with many research findings (Moyer-Packenham, et al., 2013; Suh, 2005;

Deliyianni, Michael & Pitta-Pantazi, 2006; Moyer, Niezgoda & Stanley, 2005) For example, in a study conducted at the same level, the effect of manipulatives in 3rd grade fraction teaching is similar to the study result of Suh (2005), who stated that there is no statistical difference between the performances of students who study with virtual manipulatives and those who study with concrete manipulatives. However, Brown's (2007) experimental study on fractions with primary school 6th grade students does not coincide with the result that teaching by applying concrete manipulative is more successful than teaching using virtual manipulative. This difference may be due to differences in class levels. For instance, in a study about geometry (Olkun, 2003) it is stated that fourth graders gained more in a concrete situation, while fifth graders benefited more from the virtual environments.

It is seen that there is no statistically significant difference between the motivations of the group in which concrete manipulatives are used, and the motivations of the group in which virtual manipulatives are used towards the mathematics lesson, and the motivations of the control group students. When the related literature is examined; the result that manipulatives are not effective in increasing students' mathematics motivation is similar to the result of Önver's (2019) study conducted with 5th grade students on the subject of fractions and fraction operations. In the research; manipulative support was applied to the experimental group and traditional mathematics teaching method was applied to the control group. According to the findings, it was observed that there was no difference between motivation posttest scores of both groups for mathematics motivation. Lee & Chen's (2015) view that using manipulative makes learning enjoyable can be discussed as a contradiction with this finding of the study. The fact that the motivation of the students did not differ in all three groups may be due to the fact that the study was carried out in a short time.

The findings of the study can be reinterpreted with the findings of Moyer's (2010) study in which middle school mathematics teachers stated that manipulatives are not necessary and that manipulatives cause a waste of time. In addition, as Speer (2009) stated, the reason why students experience difficulties and negative attitudes toward virtual manipulatives may be due to students not being familiar with teaching lessons with technology. In addition to certain advantages, limitations of virtual manipulatives have been stated (Litster, Moyer-Packenham, & Reeder, 2019).

Within the scope of this study, in which the effect of concrete and virtual manipulative supported fraction teaching on 3rd grade students' comprehension of fractions and their motivation towards mathematics lesson was examined; when the fraction comprehension posttest mean scores of the experimental group students and control group students were compared, a statistically significant difference was found in favor of the experimental groups. It was seen that only the use of concrete manipulative and only the use of virtual manipulative had a positive effect on fraction comprehension compared to the control group.

One of the most important results that came to the fore within the scope of the study is that an insignificant difference was found between the fraction comprehension of the group who was teaching fractions with concrete manipulative support, and the fraction comprehension of the group who was taught fraction with virtual manipulative support. In other words, concrete manipulative assisted teaching and virtual manipulative assisted teaching have no superiority in terms of both comprehension and motivation when compared to each other. Using manipulatives as complementary to each other may give better results.

A significant difference was found between the motivation of the mathematics lesson of the group in which concrete manipulatives were used and the group in which virtual manipulatives were used, and the motivation of the control group for mathematics lesson. That is, we cannot say that using concrete or virtual manipulatives in the lesson motivates the students to the math lesson.

5. Implications and Further Research

This study was conducted with an experimental method related to manipulative assisted teaching in the area of fractions unit in primary school 3rd grade mathematics lesson. Teaching using manipulative; other disciplines, other subjects of mathematics and its effects on different grade levels can be investigated.

This study was conducted by forming an experimental group using concrete manipulatives, an experimental group using virtual manipulatives, and a control group. A third experimental group, in which both virtual and concrete manipulatives are used together, can be added to the experimental groups and work on four groups can be done.

The use of manipulative in fraction teaching can be used as a teaching method that can be preferred according to the teaching approach. The purpose of this teaching approach is online learning, face-to-face learning, hybrid learning, etc. may be. In addition, it can be left to the initiative of the teacher according to the developmental level of the classroom and the flow of the subject. For example, using virtual manipulatives in asynchronous interactive lessons; the use of concrete manipulatives in face-to-face synchronous lessons can be easy and functional or concrete and virtual manipulatives can be used sequentially as complementary to each other. Regarding this; It was emphasized that both manipulatives should be used in lesson environments in a way that complements each other in order to maximize students' learning (Gülkılık, 2013; Takahashi, 2002).

It is seen that the effect of concrete and virtual manipulatives, which have been studied for many years, varies according to the class level, the math subject studied, the method of application and time. In-depth studies can be conducted with qualitative studies on the reasons for different findings.

6. Ethical Issues

All ethic rules were followed and ethical approval was obtained from the authorities.

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