Deogratias, E. (2022). Using a crossing method as an alternative approach for teaching systems of linear equations in secondary schools. *International Online Journal of Education and Teaching (IOJET)*, *9*(2). 1022-1031.

Received : 12.01.2022
Revised version received : 21.03.2022
Accepted : 25.03.2022

USING A CROSSING METHOD AS AN ALTERNATIVE APPROACH FOR TEACHING SYSTEMS OF LINEAR EQUATIONS IN SECONDARY SCHOOLS

Research article

Emmanuel Deogratias (0000-0001-9177-3524).

University of Dodoma, Tanzania

E-mails: edeo1982@yahoo.com/deograti@ualberta.ca

Correspondence (only of the corresponding author)

Biodata:

Emmanuel Deogratias is a mathematics lecturer, at the Department of Mathematics and Statistics in University of Dodoma, Tanzania. He is currently the Coordinator for Research and Publication in the Department of Mathematics and Statistics. His research interest is on mathematics education.

USING A CROSSING METHOD AS AN ALTERNATIVE APPROACH FOR TEACHING SYSTEMS OF LINEAR EQUATIONS IN SECONDARY SCHOOLS

Emmanuel Deogratias

edeo1982@yahoo.com/deograti@ualberta.ca

Abstract

This paper presents an alternative approach of teaching systems of two linear equations to the students in Tanzanian ordinary secondary schools. I conducted a desk-based research on methods that have been used by in-service mathematics teachers for teaching systems of two linear equations in ordinary secondary schools. It was found that five common methods (substitution, elimination, graphical, inverse matrix, Cramer's rule) have been used for teaching a system of two linear equations and all methods yield the same answer regardless of having different ways of approaching the system. I realized that a crossing method (alternative approach) is not found in the literature and yet not used by teachers for teaching students a system of two linear equations. But this crossing method yields similar answers with that resulted when using the five common methods. I present this alternative method in this paper by comparing with the answers obtained using five methods while focusing on two systems of two linear equations. This new approach has implications in teaching, learning and solving systems of two linear equations in ordinary secondary schools, including mathematics teachers and educators can use this method for teaching students in solving systems of two linear equations.

Keywords: Mathematics teachers, crossing method, systems of two linear equations

1. Introduction

This paper starts by introducing a crossing method as a new approach of teaching systems of linear equations in Tanzanian ordinary secondary schools. Then, the paper describes a crossing method for teaching systems of two linear equations in ordinary secondary schools. After that, the paper presents ways of solving systems of two linear equations using a crossing method by comparing the results with other methods (Substitution, elimination, graphical, inverse matrix, and Cramer's rule). The paper ends by providing concluding thoughts based on the results obtained while solving systems of two unknown linear equations using six methods (substitution, elimination, graphical, inverse matrix, Cramer's rule, and crossing) of solving the systems.

2. Introducing crossing method as a new approach of teaching systems of linear equations in Tanzanian ordinary secondary schools

Crossing method is an alternative approach that can be used for teaching students the systems of two linear equations for two unknowns in secondary schools. It is simple to use while solving systems of linear equations and takes short time to get the answer. Also, this approach yields the same answers with that obtained when teaching systems of two linear equations using elimination, graphical, substitution, Cramer's rule, and inverse matrix methods as presented in this paper.

Elimination, substitution, graphical, Cramer's rule, and inverse matrix methods are the current methods used for teaching systems of two unknown linear equations in Tanzanian mathematics curricula for secondary schools (Tanzania Institute of Education, 2005; 2009a; 2009b). Elimination, substitution, and graphical methods are taught to students when they are in Form I in their mathematics classes while inverse matrix and Cramer's rule methods are taught to the students when they are in Form IV in their ordinary secondary schools (Tanzania Institute of Education, 2005; 2009a; 2009b). All methods yield the same answer although they have different approaches of getting the answer.

This paper presents results based on desk-based research of what have been done in the literature about the methods for solving systems of two unknown linear equations in Tanzanian secondary schools and beyond secondary schools including universities, diploma colleges, and vocational trainings. From the desk-based research, it was found that five common methods (substitution, elimination, graphical, Cramer's rule, and inverse matrix) have been used for teaching systems of linear equations for two unknowns in Tanzanian ordinary secondary schools (Tanzania Institute of Education, 2005; 2009a; 2009b). Because of that this paper introduces an alternative approach (crossing method) for teaching systems of two linear equations in secondary schools. The paper is going to illustrate this alternative approach by considering a single system of two linear equations. After that, other two systems of two linear equations will be chosen as examples to present crossing method by comparing the results that will be obtained by using other five common methods (substitution, elimination, graphical, inverse matrix, Cramer's rule).

3. Describing a Crossing method for teaching systems of two linear equations

Consider the following system of linear equations:

$$\begin{cases} ax + by = A \\ cx + dy = B \end{cases}$$

By crossing the two linear equations in the system, we have:

$$(cx + dy)A = (ax + by)B$$

By opening brackets both sides, we have:

$$Acx + Aby = aBx + bBy$$

By combining like terms, we have:

$$Acx - aBx = bBy - Ady$$

By factorizing out x and y, we have:

$$(Ac - aB)x = (bB - dA)y$$

Letting $x_1 = bB - dA$ and $y_1 = Ac - aB$, we have:

$$y_1 x = x_1 y$$

Then, $x = \frac{x_1 y}{y_1}$ or $y = \frac{y_1 y}{x_1}$

But $x_1 = bB - dA$ and $y_1 = Ac - aB$. Then,

$x = \frac{(Bb-Ad)y}{Ac-aB}$ or $y = \frac{(Ac-aB)x}{Bb-Ad}$

By considering the ratios, we have:

$\frac{x}{y} = \frac{Bb-Ad}{Ac-aB}$ or $\frac{y}{x} = \frac{Ac-aB}{Bb-Ad}$

We can illustrate this method by solving the following system of linear equations:

$$\begin{cases} 5x + 2y = 8 \\ 3x - 7y = 13 \end{cases}$$

Given that $B = 13, A = 8, b = 2, a = 5, c = 3, d = -7$.

We know that $\frac{x}{y} = \frac{Bb-Ad}{Ac-aB}$ or $\frac{y}{x} = \frac{Ac-aB}{Bb-Ad}$. Then, by considering $\frac{x}{y} = \frac{Bb-Ad}{Ac-aB}$ we have:

$$\frac{x}{y} = \frac{(13)(2)-(8)(-7)}{(8)(3)-(5)(13)}$$
$$= \frac{82}{-41}$$
$$= \frac{2}{-1}$$

By considering the ratios of $\frac{x}{y} = \frac{2}{-1}$,

we have $x = 2, y = -1$ which give the solution of the system of two linear equations

$$\begin{cases} 5x + 2y = 8 \\ 3x - 7y = 13 \end{cases}$$

4. Solving systems of two linear equations using crossing method by comparing the results with substitution, elimination, graphical, and inverse matrix methods

In this section, five common methods and crossing method are used for solving the system of two linear equations:

$$\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$$

4.1.Solving the above system of linear equations by using crossing method

Given that $B = 2, A = 4, b = 1, a = 1, c = 1, d = -1$.

We know that $\frac{x}{y} = \frac{Bb-Ad}{Ac-aB}$ or $\frac{y}{x} = \frac{Ac-aB}{Bb-Ad}$. Then, by considering $\frac{x}{y} = \frac{Bb-Ad}{Ac-aB}$ we have:

$$\frac{x}{y} = \frac{(2)(1)-(4)(-1)}{(4)(1)-(1)(2)}$$
$$= \frac{6}{2}$$
$$= \frac{3}{1}$$

By considering the ratios of $\frac{x}{y} = \frac{3}{1}$,

we have $x = 3, y = 1$ which give the solution of the system of linear equations

$$\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$$

We compare this result with the results that will be obtained by using substitution, elimination, graphical, Cramer's rule and Inverse matrix methods.

4.2.Solving the system of linear equations by using the method of substitution

Let us return to the linear equations:

$x + y = 4$ (1)

$x - y = 2$ (2)

By arranging equation (1), we find that $x = 4 - y$ (3)

We can now substitute (3) into (2), we have

$$(4 - y) - y = 2$$
$4 - y - y = 2$ (opening brackets)

$4 - 2y = 2$ (adding like terms)

$4 - 2 = 2y$ (arranging like terms)

$2 = 2y$ (subtracting numbers)

$1 = y$ (dividing by 2 both sides)

Finally using equation (3), we have $x = 4 - 1 = 3$

Therefore $x = 3, y = 1$

4.3.Solving the system of linear equations by using the method of elimination
We illustrate the second method of solving a system of linear equation using similar example used in the method of substitution.

$x + y = 4$ (1)
$x - y = 2$ (2)

Since the magnitudes of the coefficients of y and x are the same in both equations. we can now add equation (1) and Equation (2), we will find y disappearing and yielding:

$$2x = 6$$

Therefore, $x = 3$
Now that we have the value for x we can substitute this value into either equation (1) or 2 in order to find the value of y. Choosing to substitute the value of x into equation (1), we have:

$3 + y = 4$ (substituting the value of x into equation (1))

$y = 4 - 3$ (collecting like terms)

$$y = 1$$

Therefore $x = 3, y = 1$

4.4.Solving a system of linear equations by using graphical method
By drawing the two lines on the same $x - y$ plane, we found that the two lines intersect at $(3,1)$. This $(3,1)$ gives the solution of the system of the linear equations such that $x = 3$ and $y = 1$. This means that the solution set is $(3,1)$. Therefore $x = 3$ and $y = 1$.

4.5.Solving the system of linear equations by using inverse matrix method
Using similar system of linear equations:

$$\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$$

We can write the system in matrix form as:

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, and $C = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$. Then, $AX = C$.
By introducing and multiplying by A^{-1} both sides of $AX = C$, we have:
$A^{-1}AX = A^{-1}C \Longrightarrow X = A^{-1}C$ which gives the solution of the system of two linear equations.
From $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, determinant $|A| = -(1)(1) - 1(1) = -2$, and the inverse of A is computed as:

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{2} \end{bmatrix}$$

Then, our solution will be:

$$X = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{2} \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{1}{2}(4) + \frac{1}{2}(2) \\ \frac{1}{2}(4) - \frac{1}{2}(2) \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

But $X = \begin{bmatrix} x \\ y \end{bmatrix}$, therefore $x = 3, y = 1$.

4.6.Using Cramer's rule to solve the same system of two linear equations

$$\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$$

We can write the system into matrix form as:

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

We convert the system into augmented matrix:

$$\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 1 & -1 & 2 \end{array}\right]$$

Let $D = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$. Then, we determine the determinant of the square coefficient matrix D from the left-hand side of the augmented matrix:

Determinant, $|D| = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = (1)(-1) - (1)(1) = -2$.

$$|D_x| = \begin{vmatrix} 4 & 1 \\ 2 & -1 \end{vmatrix} = (4)(-1) - (2)(1) = -6$$

$$|D_y| = \begin{vmatrix} 1 & 4 \\ 1 & 2 \end{vmatrix} = (2)(1) - (1)(4) = -2$$

We finally find the solution for each variable in the linear equations:

$$x = \frac{|D_x|}{|D|} = \frac{-6}{-2} = 3$$

$$y = \frac{|D_y|}{|D|} = \frac{-2}{-2} = 1$$

Therefore, $x = 3, y = 1$.

We can consider the second example by working on the following system of linear equations with two unknowns:

$$\begin{cases} 2x + 3y = 15 \\ x - 2y = 4 \end{cases}$$

4.7.Solving the above system of linear equations by using crossing method

Given that $B = 4, A = 15, b = 3, a = 2, c = 1, d = -2$.

We know that $\frac{x}{y} = \frac{Bb-Ad}{Ac-aB}$ or $\frac{y}{x} = \frac{Ac-aB}{Bb-Ad}$. Then, by considering $\frac{x}{y} = \frac{Bb-Ad}{Ac-aB}$ we have:

$$\frac{x}{y} = \frac{(4)(3) - (15)(-2)}{(15)(1) - (2)(4)}$$

$$= \frac{42}{7}$$
$$= \frac{6}{1}$$

By considering the ratios of $\frac{x}{y} = \frac{6}{1}$, we have $x = 6, y = 1$ which give the solution of the system of linear equations

$$\begin{cases} 2x + 3y = 15 \\ x - 2y = 4 \end{cases}$$

We compare this result with the results that will be obtained by using substitution, elimination and graphical methods.

5. Solving the system of linear equations by using the method of substitution

Let us return to the linear equations:

$2x + 3y = 15$ (4)

$x - 2y = 4$ (5)

By arranging equation (1), we find that $x = 4 + 2y$ (6)

We can now substitute (6) into (4), we have

$$2(4 + 2y) + 3y = 15$$
$$8 + 4y + 3y = 15 \text{ (opening brackets)}$$
$$8 + 7y = 15 \text{ (adding like terms)}$$
$$7y = 15 - 8 \text{ (arranging like terms)}$$
$$7y = 7 \text{ (subtracting numbers)}$$
$$y = 1 \text{ (dividing by 7 both sides)}$$

Finally using equation (6), we have $x = 4 + 2(1) = 6$

Therefore $x = 6, y = 1$

4.8.Solving a system of linear equations using the method of elimination

We illustrate the second method of solving a system of linear equation using similar example used in the method of substitution.

$2x + 3y = 15$ (4)

$x - 2y = 4$ (5)

Since the magnitudes of the coefficients of y and x are not the same in both equations. we can multiply by 2 in equation (5) and then subtracting equation (4) and Equation (5), we will find x disappearing and yielding:

$$7y = 7$$

Therefore, $y = 1$.

Now that we have the value for y we can substitute this value into either equation (4) or (5) in order to find the value of x. Choosing to substitute the value of y into equation (5), we have:

$x - 2(1) = 4$ (substituting the value of y into equation (5)

$x = 4 + 2$ (collecting like terms)

$$x = 6$$

Therefore $x = 6, y = 1$.

4.9.Solving a system of linear equations using graphical method

By drawing the two lines on the same $x - y$ plane, we found that the two lines intersect at $(6,1)$. This $(6,1)$ gives the solution of the system of the linear equations such that $x = 6$ and $y = 1$. This means that the solution set is $(6,1)$. Therefore $x = 6$ and $y = 1$.

4.10. Solving the system of two linear equations using inverse matrix method

Using similar system of linear equations:

$$\begin{cases} 2x + 3y = 15 \\ x - 2y = 4 \end{cases}$$

We can write the system in matrix form as:

$$\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 4 \end{bmatrix}$$

Let $A = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, and $C = \begin{bmatrix} 15 \\ 4 \end{bmatrix}$. Then, $AX = C$.

By introducing and multiplying by A^{-1} both sides of $AX = C$, we have:

$A^{-1}AX = A^{-1}C \Longrightarrow X = A^{-1}C$ which gives the solution of the system of two linear equations.

From $A = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$, determinant $|A| = -(2)(2) - 1(3) = -7$, and the inverse of A is computed as:

$$A^{-1} = \frac{1}{-7} \begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{2}{7} & \frac{3}{7} \\ \frac{1}{7} & \frac{-2}{7} \end{bmatrix}$$

Then, our solution will be:

$$X = \begin{bmatrix} \frac{2}{7} & \frac{3}{7} \\ \frac{1}{7} & \frac{-2}{7} \end{bmatrix} \begin{bmatrix} 15 \\ 4 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{2}{7}(15) + \frac{3}{7}(4) \\ \frac{1}{7}(15) - \frac{2}{7}(4) \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

But $X = \begin{bmatrix} x \\ y \end{bmatrix}$, therefore $x = 6, y = 1$.

4.11. Using Cramer's rule to solve the same system of two linear equations

$$\begin{cases} 2x + 3y = 15 \\ x - 2y = 4 \end{cases}$$

We can write the system in matrix form as:

$$\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 4 \end{bmatrix}$$

We convert the system into augmented matrix:

$$\left[\begin{array}{cc|c} 2 & 3 & 15 \\ 1 & -2 & 4 \end{array}\right]$$

Let $D = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$. Then, we determine the determinant of the square coefficient matrix D from the left-hand side of the augmented matrix:

Determinant, $|D| = \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = (2)(-2) - (1)(3) = -7.$

$$|D_x| = \begin{vmatrix} 15 & 3 \\ 4 & -2 \end{vmatrix} = (15)(-2) - (4)(3) = -42$$

$$|D_y| = \begin{vmatrix} 2 & 15 \\ 1 & 4 \end{vmatrix} = (2)(4) - (1)(15) = -7$$

We finally find the solution for each variable in the linear equations:

$$x = \frac{|D_x|}{|D|} = \frac{-42}{-7} = 6$$

$$y = \frac{|D_y|}{|D|} = \frac{-7}{-7} = 1$$

Therefore, $x = 6, y = 1$.

5. Conclusions

Substitution, elimination, graphical, Cramer’s rule, inverse matrix, and crossing methods have used in this paper as methods that mathematics teachers can use to teach students how to solve systems of linear equations for two unknowns. It was found that all methods yield similar results including an alternative approach (crossing method). Because of that this paper recommends using crossing method as an alternative way of teaching students the systems of two linear equations in ordinary secondary schools in Tanzania and beyond the country. This is important to add knowledge on teaching and solving systems of two linear equations to the existing five methods (substitution, elimination, graphical, Cramer’s rule and inverse matrix) in Tanzanian mathematics textbooks and syllabi for ordinary secondary schools.

References

Tanzania Institute of Education. (2005). *Basic mathematics syllabus for secondary schools*. Dar es Salaam, TZ: Author.

Tanzania Institute of Education. (2009a). *Secondary basic mathematics book two*. Dar es Salaam, TZ: Education Books Publishers.

Tanzania Institute of Education. (2009b). *Secondary basic mathematics book four*. Dar es Salaam, TZ: Education Books Publishers.